TP-5170E

M.V. ARCTIC

ICE TRANSITING PERFORMANCE

CONTINUOUS MODE

Prepared for

TRANSPORTATION DEVELOPMENT CENTRE
TRANSPORT CANADA

By

ALBERY, PULLERITS, DICKSON
& ASSOCIATES

In association with

COLDSTREAM MARITIME INC.
GASTOPS LTD.
D.F DICKINS ASSOCIATES LTD.

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May 1984

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17. Abstract

This report presents an analysis of data relating to the ice transiting performance of the M.V. Arctic, operating in the continuous mode in level ice. The data was acquired over the years 1979-81 during commercial voyages and dedicated trials, by means of manual measurements and a system of instruments recording directly on tapes. From the total data available the useful data was extracted and analyzed to produce an equation representing the performance of the M.V. Arctic in level sheet ice. This equation was tested against the 1974 model tests and correlation was found to be reasonably good. An indication was also obtained of the performance in concentrations of ice floes and broken channels, although the data was inadequate to derive formulae for these conditions.

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17. Résumé

Ce rapport analyse les données relatives à la tenue de l'*Arctic* se déplaçant dans des glaces uniformes, en régime de marche continue. Ces données ont été recueillies entre 1979 et 1981 au cours de voyages commerciaux et d'essais spécialisés. Pour cela, des moyens tant manuels qu'automatiques (enregistrements sur rubans) ont été utilisés. Les données exploitables ont été extraites des données brutes et analysées; il en est ressorti une équation représentant la tenue de l'*Arctic* dans des glaces uniformes. Cette équation a été confrontée aux résultats des essais sur modèle réduit faits en 1974 et la corrélation obtenue a été jugée relativement bonne. Des valeurs indicatives ont pu également être tirées sur la tenue dans diverses concentrations de floes et dans des chenaux ouverts, l'insuffisance des données empêchant la mise en équation de ces conditions.

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M.V. ARCTIC

ICE TRANSITING PERFORMANCE - CONTINUOUS MODE

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GLOSSARY OF SYMBOLS

The symbols used in this study have been standardized as far as possible to agree with definitions most commonly accepted and are not necessarily those of the original authors of the various equations to which reference is made. Special symbols not included in the glossary or special meanings given to symbols are defined where they occur.

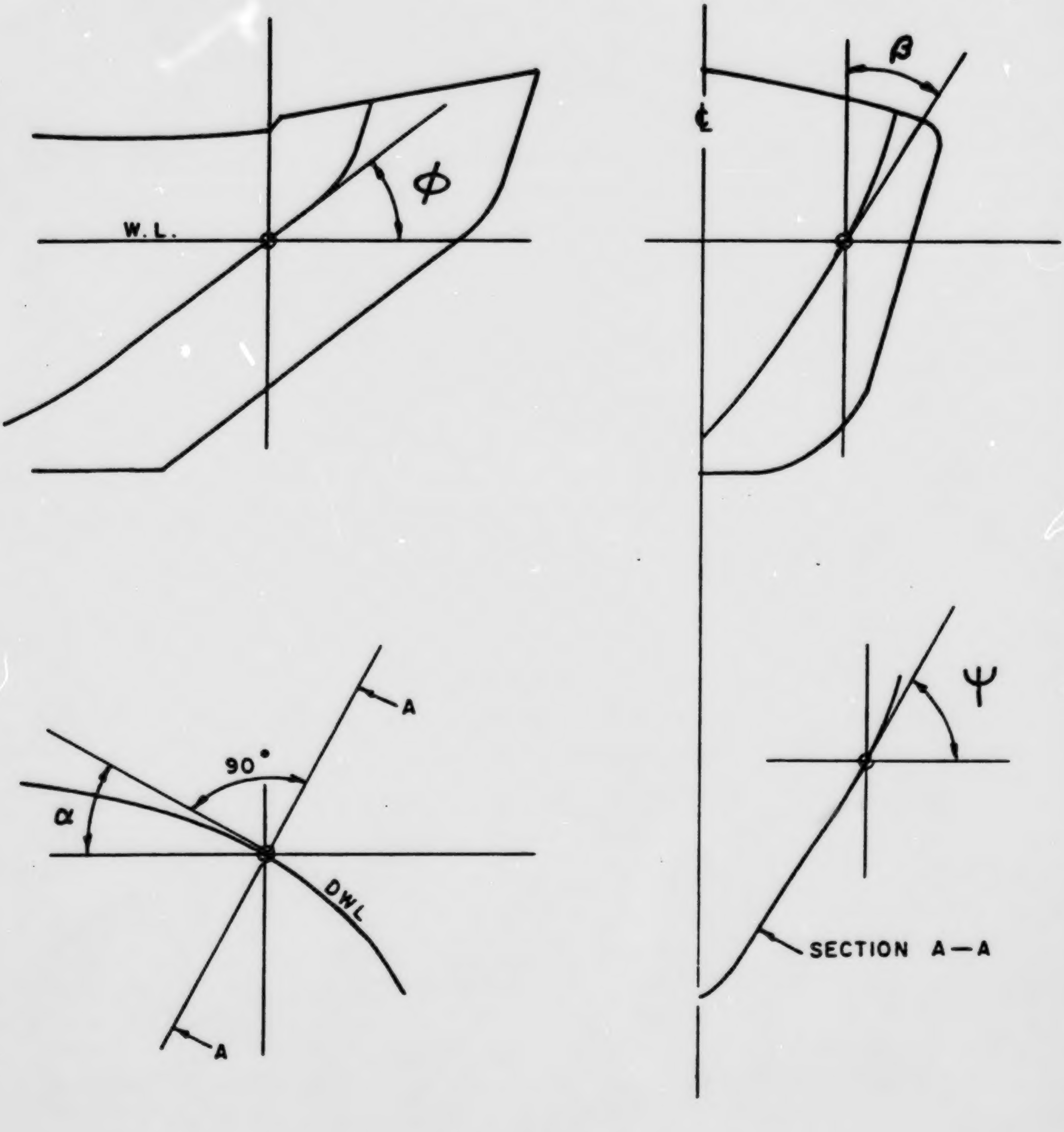
Roman Symbols	Definition	Dimensions
B	Beam of ship	L
C_o, C_1, C_2	Coefficients as defined in text	-
C_b	Coefficient of resistance due to breaking ice	-
C_s	Coefficient of resistance due to submerging ice	-
C_v	Coefficient of resistance due to velocity	-
C_B	Block coefficient	-
D	Propeller diameter	L
E	Elastic or total strain modulus	$L^{-1}MT^{-2}$
F	Froude number	-
F_h	Ice thickness Froude number, $V/\sqrt{gh}$	-
F_L	Ship length Froude number, $V/\sqrt{gl}$	-
H	Draft of ship	L
H_{BW}	Height of bow wave	L
K_T	Thrust coefficient	-
K_Q	Torque coefficient	-
L_{OA}	Length of ship, overall	L
L_{WL}	Length of ship, at waterline	L
L_{PP}	Length of ship, between perpendicular	L
L_F	Length of bow or forebody	L
L_W	Cube root of Volumetric Displacement	L
N_s	Shaft speed, RPM	$Rev.T^{-1}$
P_s	Power (Shaft)	L^2MT^{-3}
Q	Torque	L^2MT^{-2}

Roman Symbols	Definition	Dimensions
R_1, R_2	Resistance as defined in text	LMT^{-2}
R_t	Total resistance due to ice and water	LMT^{-2}
R_i	Resistance due to ice	LMT^{-2}
R_b	Resistance due to breaking ice	LMT^{-2}
R_s	Resistance due to submerging ice	LMT^{-2}
R_v	Resistance due to velocity	LMT^{-2}
R_{ow}	Resistance in open water	LMT^{-2}
T	Total thrust	LMT^{-2}
T_p	Propellers thust	LMT^{-2}
T_D	Duct thrust (for ducted propellers)	LMT^{-2}
V	Speed of ship	LT^{-1}
W	Displacement of ship, tonnes force	LMT^{-2}
Z	Number of Propellers	-
f	Friction factor as defined	-
f_d	Coefficient of dynamic friction	-
g	Acceleration due to gravity	LT^{-2}
h	Ice thickness	L
h_s	Snow depth over ice	L
ℓ	Length of ice beam or broken cusp	L

Greek Symbols	Definition	Dimensions
α	Angle of bow to centre line of ship at water surface (see page xiii)	-
α_o	Half bow entrance angle at water surface	-
β	Flare angle of transverse body section at waterline level (see page xiii)	-
γ	Weight density, as defined in text	$ML^{-2}T^{-2}$
γ_w	Weight density of water	$ML^{-2}T^{-2}$

Greek Symbols	Definition	Dimensions
γ_i	Weight density of ice	$ML^{-2}T^{-2}$
γ_Δ	Weight density of water less ice	$ML^{-2}T^{-2}$
δ	Angle as defined in text	-
η	Friction form coefficient, as defined in text	-
η_2	Shimansky bow form coefficient (ice cutting)	-
θ	Angle as defined in text	-
λ	Ratio: Full-scale/model scale	-
μ	Friction form coefficient, as defined in text	-
μ_o	Shimansky bow form coefficient (ice breaking)	-
ν	Poisson's ratio	-
ρ	Mass density, as defined in text	ML^{-3}
ρ_w	Mass density of water	ML^{-3}
ρ_i	Mass density of ice	ML^{-3}
ρ_Δ	Mass density of water less ice	ML^{-3}
σ	Strength of ice, flexural where not defined	$L^{-1}MT^{-2}$
σ_f	Flexural strength of ice	$L^{-1}MT^{-2}$
ϕ	Vertical angle between stem (or buttock) and water plane (see page xiii)	-
ψ	Vertical angle of bow sides to water plane (see page xiii)	-

DEFINITION OF BOW ANGLES



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1.0 INTRODUCTION

The M.V. Arctic is an Arctic Class II bulk carrier of 28,000 DWT which has been instrumented with a computerized data acquisition system to record its performance in ice. Data have been acquired over three operating seasons 1979-81 not only during commercial voyages but also during certain dedicated trials off Baffin Island and in the Saguenay River and Lake Melville.

The Transportation Development Centre (TDC) has arranged for a number of studies to evaluate the recorded data and to determine the ship's structural and ice transiting performance. This report deals only with the M.V. Arctic's performance in the continuous mode when transiting level sheet ice, broken channels and ice floes in various concentrations. Ice transiting performance in the ramming mode, manoeuvring performance and structural performance are the subject of separate studies.

The characteristics of the M.V. Arctic are shown in Table 1.1.

The continuous mode studies were carried out by Albery, Pullerits, Dickson & Associates with the assistance of the following firms as sub-consultants:

Coldstream Maritime Inc.
Specialists in naval architecture and
marine research including the performance
of ships in open water and ice.

D.F. Dickins Associates Ltd.
Specialists in arctic transportation
and sea ice dynamics.

GasTops Ltd.
Specialists in propulsion systems
engineering and computer analysis.

TABLE 1.1

M.V. ARCTIC - SHIP CHARACTERISTICS

Length Overall	209.55 m
Length Between Perpendiculars	196.60 m
Moulded Breadth (Beam)	22.86 m
Moulded Depth	15.24 m
Moulded Draught	10.97 m
Summer Draught	10.82 m
Block Coefficient	0.753
Prismatic Coefficient	0.761
Light Ship	9,276.1 t
Displacement at 10.81 m draught	37,724.1 t
Dead Weight at 10.81 m draught	28,448.0 t
Cargo Capacity	34,448.0 cubic metres
Propulsion Machinery - MAN Diesel	11,014 kilowatts
Generators	900 kW x 3
Complement	40
Speed on Trials	15.5 knots
Cruising Range	14,000 nautical miles

2.0 REVIEW OF EXISTING ICE RESISTANCE RESEARCH

2.1 General

It is generally possible to produce an equation for the resistance of ice to the motion of a ship using regression analysis which will represent the data from full scale or model tests on a particular ship operating in a particular range of ice conditions. However, such a formula or equation is applicable only to that ship in the range of ice conditions tested and cannot be extrapolated to other ships of different size or shape or to different ice conditions, or even to significantly different speeds.

It is therefore desirable that any equation should be based on a wide range of tests of different vessels and on a reasonable analytical approach to the fundamental forces and energy involved in transiting level ice of various types. For this purpose it is helpful to review the history of attempts to derive equations for the resistance of ice to the motion of a ship and such a review is given in Appendix A.

The study terms of reference require resistance formulations to be identified and evaluated for three types of ice conditions:

(a) continuous level sheet ice
(b) pack ice or concentrations of ice floes
(c) broken channels.

Historically most research and tests have been carried out on the resistance of continuous sheet ice and comparatively little information is available on the resistance of broken channels and ice floes in various concentrations. For this reason the following commentary concentrates mainly on the resistance of continuous sheet ice.

2.2 Resistance of Continuous Level Sheet Ice

As shown in the historical review given in Appendix A, the early tendency was to attribute resistance almost entirely to the force required to break the ice and to consider friction as an essential part of this force. This is not surprising considering the size of the vessels and the small speeds involved at the time.

As a result of the work of Kashtelyan, there has been a general acceptance of the thesis that the resistance of ice to the motion of a ship is made up of three main components:

- Resistance due to breaking ice, independent of velocity.
- Resistance due to submerging broken ice.
- Velocity dependent resistance.

However, within this grouping of forces there has not been agreement in the past as to how the resistance varies with the ice thickness, h, and on what power of V the velocity resistance depends. This is shown in Table 2.1 by comparison of selected equations put forward up to 1976. The wide divergence of results obtained by extrapolating some of these equations to large bulk carriers in various ice thicknesses is shown in Figures 2.1 and 2.2.

For the comparison of model and full scale results and also for confidence in extrapolation to larger ships it is desirable to have a general equation which can be put in the form of a dimensionless grouping of parameters such as the following, which is commonly accepted:

$$\frac{R}{\rho_w g\, B h^2} = C_0 + C_1 \frac{\sigma}{\rho_w g h} + C_2 \left(\frac{V}{\sqrt{gh}}\right)^n$$

TABLE 2.1

COMPARISON OF SELECTED EQUATIONS TO 1976

	SOURCE	ICE BREAKING	ICE SUBMERSION	VELOCITY DEPENDENT
		$C_b B\sigma h\mu_o$	$C_s Bh^2 \rho_i g \mu_o$	$C_v B^x hV^y \frac{1}{\eta_2}$
1	Kashtelyan	$0.0040 B\sigma h\mu_o$	$3.6 Bh^2 \rho_i g \mu_o$	$0.25 B^{1.65} \frac{hV}{\eta_2}$
2	Assur	$0.0033 B\sigma h\mu_o$	$3.0 Bh^2 \rho_i g \mu_o$	$1.44 \frac{B^{1.67}}{B_o^{0.67}} h \frac{V}{V_o} \frac{1}{\eta_2}$
3a	Lewis & Edwards	$5.46 \rho_w g Bhsc$	$5.26 Bh^2 \rho_w g$	$4.35 \rho_w Bh^2 g \frac{V}{\sqrt{gh}} + 1.67\rho_w BHV^2$
3b		$0.05 B\sigma h$	$4.24 Bh^2 \rho_w g$	$8.95 \rho_w Bh^2 g \frac{V}{\sqrt{gh}}$
3c		$C_{bv} B\sigma h \frac{V}{\sqrt{gh}}$	$C_s Bh^2 \rho_w g$	$C_v \rho_w Bh^2 g \frac{V^2}{gh}$
3d		$C_b B\sigma h$ WHERE: $C_b = -0.032 + 0.26f + 0.39 C_B)$	$C_s Bh^2 \rho_w g$ WHERE: $C_s = 5.7 + 0.15 \frac{H}{h} - 7.8 C_B^2$	$C_v \rho_w Bh^2 g \frac{V^2}{gh}$ WHERE: $C_v = (-68.16 + \frac{0.048\sigma}{\rho w g h})$ $+ 224 C_B - 181 C_B^2 + 0.25 \frac{H}{h}$
4	Enkvist	$2C_b Bh \rho_\Delta gH$ WHERE: $C_b = 5.8 \frac{\sigma}{E} \times \frac{\sigma C}{g\rho_\Delta H}$ (C' for bow shape)	$2C_s B \rho_\Delta gHh$ WHERE: $C_s = \eta_{sp} + \frac{L_E}{H} \mu\eta_{s\mu E}$	$C_i\rho_i BV^2 + C_2 B\ell g\rho_w$ WHERE: $C_1 = C' (h + 0.213\ell)$ $C_2 = C'' (0.17\ell \sin \Psi + H_{BW})$
5	Mathews	$C_b B\sigma h$ WHERE: $C_b = 0.03$	$C_s Bh^2 \rho_w g$ WHERE: $C_s = 6 \pm 50\%$	$C_v \rho_w BhV^2$ WHERE: $C_v = 6 \pm 50\%$
6	CCGS	$\sigma h^2 (0.1B(10)^{0.147V} + 0.0085L_F)$		
7	APD	$0.042\sigma Bh$	$2.96 Bh^2 \rho_w g$	$1.8 C_s\rho_w Bh^{0.7} H^{0.3} V^2$
8	Kloppenburg	$C_b\sigma h^2$	$C_s Bh^2 \rho_w g$	$C_v\rho_w BhV^2$
9	Vance	$C_b B\sigma h$	$C_s Bh^2 \rho_\Delta g$	$C_v\rho_w V^2 LH^{0.65} B^{0.35}$

F[illegible]efi[illegible]on [illegible]ymb[illegible] see [illegible]oss[illegible]

Where C_0, C_1 and C_2 are coefficients to be determined and the other symbols are as defined in the Glossary of Symbols given after the Table of Contents at the beginning of this report.

However, the selection of parameters forming the dimensionless ratio is still a matter of judgement. For instance, the draft of the vessel, H, is not included in the above equation but the 1975 model tests on the Polar VII[3] icebreaker strongly suggest that resistance increases with draft. There is also a general assumption that the Froude number, F, in the velocity dependent term should be an ice thickness Froude number, $V/\sqrt{gh}$, rather than one involving a dimension of the vessel, such as draft, $V/\sqrt{gH}$ or a combination of both. In any case, there is no agreement on whether the velocity resistance should be dependent on F or on F^2. Finally, the modulus of strain of the ice, E, is not included in the icebreaking term, even though it is generally agreed that the energy of breaking involves the deflection of the ice in bending.

There is obviously a large variety of combinations of parameters and indices that will satisfy the dimensionless equation and certainly some evidence that there might be a better form than that commonly accepted. In the discussion on Vance's paper[11] in the Symposium on Icebreaking and Related Techniques, 1975, Dr. F.J. Legerer, after noting the discrepancies between the results of various researchers, did in fact, question whether another class of mathematical model should be tried.

Another assumption implicitly made in nearly all resistance equations is that the ice is always broken in bending and that only flexural strength is important. This too was questioned by Dr. Legerer at the Ice Tech, '75 symposium, who argued that "one may expect failure by bending in thin ice only and with increasing thickness a combination of bending and shear until almost pure shearing occurs". That there may be some validity to this comment, particularly in relation to the hull shape and friction, is discussed later.

2.2.1 Influence of Ice Strength

The range of flexural ice strength and some other properties of ice measured or adopted by various authorities is shown in Table 2.2. There is a fair range of flexural strength for saline ice as might be expected in view of its variation with salinity and temperature. It might be thought, therefore, that the assumption of the correct flexural strength would be important to obtain a reasonably accurate prediction of the resistance of ice to the motion of a ship. In fact, until recently, authorities such as Kashtelyan[1] and Enkvist[12] have maintained that ice strength has little effect on the total resistance to motion. However, recent full-scale and model research has shown that the breaking component of resistance and the ice strength is much more important than previously supposed. (Refs. 22, 24 and 28).

The icebreaking resistance is not a true force but the time integral of a series of energy expenditures divided by distance. As such, it does not vary only with the strength of the ice, σ, but with:

$$\frac{\sigma}{E} \times \sigma$$

where E is the total strain modulus involving plastic as well as elastic deformation. The strain modulus will vary with the strength of the ice, its thickness, salinity and temperature and with the rate of loading. Insofar as E may not vary in the same proportion as σ, an equation omitting the ratio σ/E will be in error. The possibility that E may change more rapidly than σ, particularly in the case of model ice, may account for the apparent observation that the strength of the ice has insignificant effect on resistance.

2.2.2 Influence of Vessel Shape and Friction

The influence of bow shape and friction on the performance of an icebreaker has long been recognized but there is no general agreement on the interaction of shape and friction and which of the three components of resistance are mainly affected by friction.

TABLE 2.2

Range of Properties of Saline First Year Ice

Source	Strength Crushing kPa	Ultimate Bending kPa	Strength Tension kPa	Strength Shear kPa	Young's Modulus kPa
Kashtelyan[1]		240- 1300			2900
Korzhavin[1]	1200 2900			145- 585	1500* 2000
Enkvist[12]		200- 500			1000-* 2500
Butkovitch[46]	2100 4200				
Weeks & Assur[47]		620			
German & Lawrence[48]		248 520			
Federking[49]					7500*
Dykins[50]		275- 690	400- 860		
Traetteberg et. al.[51]					4000- 9000
Edwards, et. al.[2]		275- 485			
Kohnen[53]		520			5260
Frederking & Hausler[54]		200- 640			1300- 3170
Michailidis & Murdey[18]		330			2450
McIntosh & Welsh[20]	765	254			

A great deal of model testing has been undertaken to find the best shape and angle of the ice-cutting stem so as to combine effective icebreaking with reasonable open water performance. Some of the numerous stem shapes are shown in Figure 2.3. It has become apparent, however, that until fairly recently, insufficient attention has been given to the vertical angle of the shoulders of the vessel where the bow is reaching its maximum beam.

From the geometry of the icebreaking resistance it can be shown that the ice cannot in theory be broken in bending by the sides of the bow where the flare angle is less than $\tan^{-1} f$ and f is the coefficient of friction. The ice will therefore have to be broken by shear, buckling or crushing thus imposing much greater lateral forces and frictional resistance on the vessel. This question of shoulder friction, which is discussed in more detail in Section 9, does not appear to have been adequately appreciated until recently.

In the past most authorities, such as Kashtelyan and Enkvist, attributed frictional resistance mainly to the action of ice pieces on the hull of the vessel after breaking as part of the submergence resistance. However, in recent years the importance of shoulder resistance has been recognized and has led to the spoon shaped ice-breaking bow as adopted for the icebreaker Kigoriak and the Waas concept described by Freitas.[23]

Since it may not always be possible to have a fairly large vertical angle as the forebody reaches maximum beam it may still be desirable to provide a ship surface of minimum ice friction in critical areas. The search for low friction coatings has been proceeding for several years and was first described by Makinen[14] in Ice Tech 75. It is now possible to apply a solvent free epoxy coating which will greatly reduce friction and which has a remarkable resistance to wear in ice navigation. Bubbler systems may also be used to reduce friction and are claimed to reduce ice resistance up to 32%.

Some typical values of dynamic friction are given in Table 2.3. Static friction may be considerably higher.

2.2.3 Influence of Vessel Dimensions

Nearly all those who have studied ice resistance have concluded that all three main components of resistance vary directly with the beam of the ship. One notable exception is Kloppenburg[9] who has concluded that the icebreaking term should vary with σh^2 rather than σhB and in the discussion on his paper put forward a detailed argument for this contention.

There is general agreement that the length of the ship astern of the forebody does not affect the resistance except in a pressure field. Enkvist does, however, introduce the length of the forebody L_E in connection with frictional resistance of submerging the broken ice. In the previous section it is suggested that L_E should also be introduced in the icebreaking component.

Of the equations listed in Table 2.1 only Equation 4 by Enkvist and Equation 3d by Edwards and others introduce the draft of the ship, H. The model tests on the Polar VII icebreaker[3] produced strong evidence that draft is a factor both in the resistance due to submersion of ice and also in the velocity dependent resistance. Further tests would appear to be desirable to confirm this finding but some dependence on draft would seem to be logical.

2.2.4 Validity of Model Tests

For a model to be able to predict the performance of a full-size ship, the principles of dimensional similarity must be satisfied. To do this completely in the case of ship models means satisfying simultaneously the laws of Cauchy, Froude and Reynolds. Kashtelyan[1] proves that the similarity criteria for Cauchy and

TABLE 2.3

DYNAMIC COEFFICIENT OF FRICTION BETWEEN STEEL & ICE

	Snow Covered	Ice Only	Comments
Kashtelyan[1]	0.12 - 0.26	0.08 - 0.13	
Makinen, et. al.[14]		0.04 - 0.18	Higher extreme
Enkvist[12]	0.1 - 0.3	0.03 - 0.11	
Enkvist[28]		0.16	
Ryvlin & Petrov		0.01 - 0.20	
Airaksinen	0.066 - 0.307	0.025 - 0.109	
Vance et al[15]	0.165 - 0.170	0.05 - 0.15	
Comfort[15] Discussion		0.16 - 0.26	Bare Steel
		0.07 - 0.11	Painted

DYNAMIC COEFFICIENT OF FRICTION BETWEEN INERTA 160 & ICE

Vance et al[15]	0.085 - 0.145	0.015 - 0.045	
Voelker et al[27]		0.031 - 0.056	Continuous Sliding
Voelker et al[27]		0.082 - 0.087	Impact & Crushing
Enkvist[28]		0.08	

Froude are essentially the same but it is not possible to simultaneously satisfy the Reynolds criterion which is applicable to some portions of the velocity dependent resistance. However, as with ordinary ship modelling in open water, it is possible to separate out the forces dependent on the Reynolds criterion and, if these can be calculated or modelled separately, the necessary adjustments can be made. Model tests in ice are therefore made to satisfy the Cauchy and Froude criteria which require dimensional relationships as shown in Table 2.4.

All early model tests and many current tests are made using a synthetic ice generally made from paraffin wax. However, paraffin wax tends to be too plastic and, although new forms of synthetic ice have been developed, notably by Dr. Bernard Michel at Laval University, there is a growing tendency to use saline ice specially formed in a refrigerated testing facility. A list of such facilities now available is given in Reference 43.

The strength of ice decreases with salinity and, in order to produce model ice with the required strength of σ/λ, its salinity is greatly increased beyond that of real sea ice and careful procedures have to be adopted to form it successfully. Even so, as clearly demonstrated by Enkvist[12], the model ice is much more plastic than real ice and the energy of breaking model ice is much greater than it should be for the correct model strength and thickness. The bending strain modulus of ice decreases with salinity more quickly than the strength and it is therefore virtually impossible to obtain the same ratio of σ/E for model saline ice as for real sea ice. This tends to introduce significant errors in the icebreaking component of the model resistance. To complicate the matter further, friction is now agreed to be an important force and the exact relationship of friction to the icebreaking force and to the total resistance has yet to be established. However, the recent introduction of carbamide ice which is claimed to satisfy the similarity laws for ice modelling may solve some of the present problems.[37]

TABLE 2.4

Scaling Laws Applicable to Modelling Icebreaking and Ice-Resistance Characteristics of Ice-Breakers

$L/L_m = \lambda$	WHERE:	L = Linear dimensions
$I/I_m = \lambda^5$		I = Mass Moment of Inertia
$v/v_m = \lambda^{\frac{1}{2}}$		v = Ship Speed
$\nu/\nu_m = 1$		ν = Poisson Ratio
$\rho/\rho_m = 1$		ρ = Mass Density of Water
$\rho i/\rho i_m = 1$		ρi = Mass Density of Ice
$M/M_m = \lambda^3$		M = Mass of Ship
$R/R_m = \lambda^3$		R = Resistance force due to motion of ship in ice only.
$E/E_m = \lambda$		E = Modulus of elasticity of ice.
$\sigma/\sigma_m = \lambda$		σ = Flexural strength of ice.
$h/h_m = \lambda$		h = Ice thickness.
$f_d/f_{dm} = 1$		f_d = Dynamic ice friction coefficient.
$P/P_m = \lambda^{7/2}$		P = Power

Subscript m = model scale
No Subscript = full scale

SOURCE: Ref. No. 12

NOTE: The above scaling laws are based on keeping the Froude number $\frac{V^2}{gL}$ constant where L is a linear dimension. Forces dependent on the Reynolds number $\frac{VL}{k}$ are considered to be insignificant. k is the kinematic viscosity and equal to viscosity/mass density. The Cauchy number, $\frac{V^2E}{p}$, is also involved in model scaling.

At the present state of the art, there is some reason to doubt the ability of model tests to predict accurately the resistance of ice to the motion of a ship. On the other hand, there is no doubt of the value of model tests for qualitative comparison of bow shapes and general parametric analysis. One thing is certain. Until a more fundamentally correct general equation for ice resistance has been developed, the coefficients and indices produced from a particular series of model tests by regression analysis cannot be used with any confidence for extrapolation outside the range of data and assumptions used in the model tests.

2.2.5 Conclusion

In conclusion, it may be said that the science of predicting the resistance of ice to the motion of a ship is far from perfected. Equations can be produced, from model or full-scale tests to predict the performance of a particular ship within the range of the test data but a completely acceptable general predictor equation has yet to be developed.

2.3 Resistance in Pack Ice and Broken Channels

As far as pack ice and broken channels are concerned, Kashtelyan remains the authority for the prediction of resistance in broken ice.

The mathematical expressions presented by Kashtelyan are empirical, based on full scale data and model data.

The general expression presented by Kashtelyan for calculation of the ice resistance is given below.

$$R_i = \gamma_i \sqrt{rh} \left(\frac{B}{2} \right)^2 \left[K_1 \left(1 + 2 f_d \alpha_\mu \frac{L}{B} \right) + K_4 f_d \alpha \frac{L}{B} S_c \right] + K_2 \gamma_i rhB \left(f_d + \alpha_\mu \tan \alpha_0' \right) \frac{V}{\sqrt{gL}} + K_3 \gamma_i rhL \tan^2 \alpha_0' \frac{V^2}{gL}$$

Where r - length of finely broken ice, m

α - ice waterplane coefficient

α_μ - ice waterplane coefficient of the bow part of the ship

α_0' - entering angle of bow part of the ice waterline

S_c - compression force in ice

and K_1, K_2, K_3, K_4 - dimensionless coefficients determined from data of model experiments.

The coefficients should depend on the degree of packing of ice and on the width of the channel filled with finely broken ice.

A much simplified purely empirical expression derived from data of full-scale measurements was used by Kashtelyan for prediction of ice resistance in naturally broken channels.

This expression is presented in Section 5.6.

Reference to full scale trials to determine the resistance to ships following a channel broken by an icebreaker is given in Appendix B. Ref. Tsoy [38].

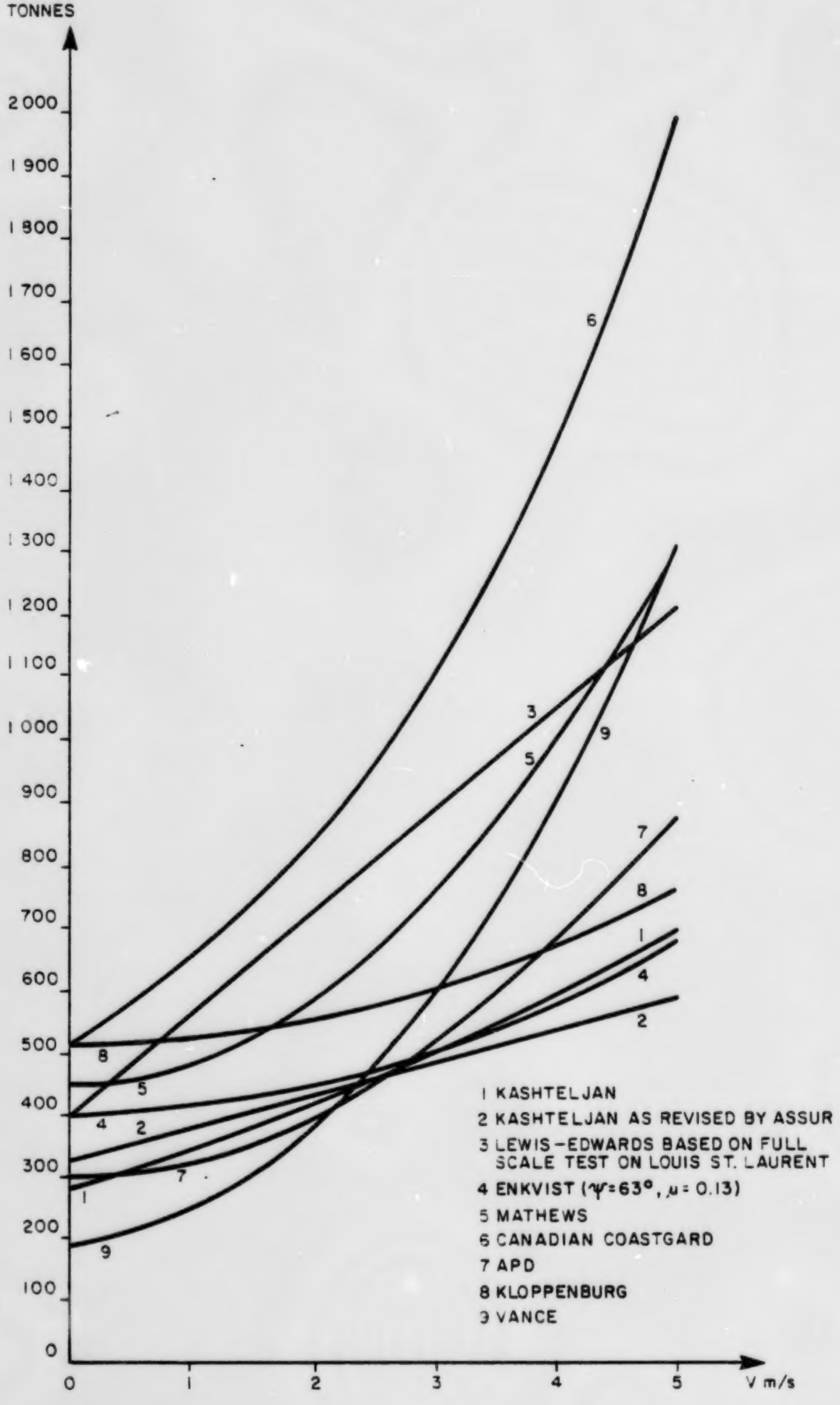


FIGURE 2.1 ICE RESISTANCE APPLIED TO 300,000 DWT VLCC IN 1.0m ICE

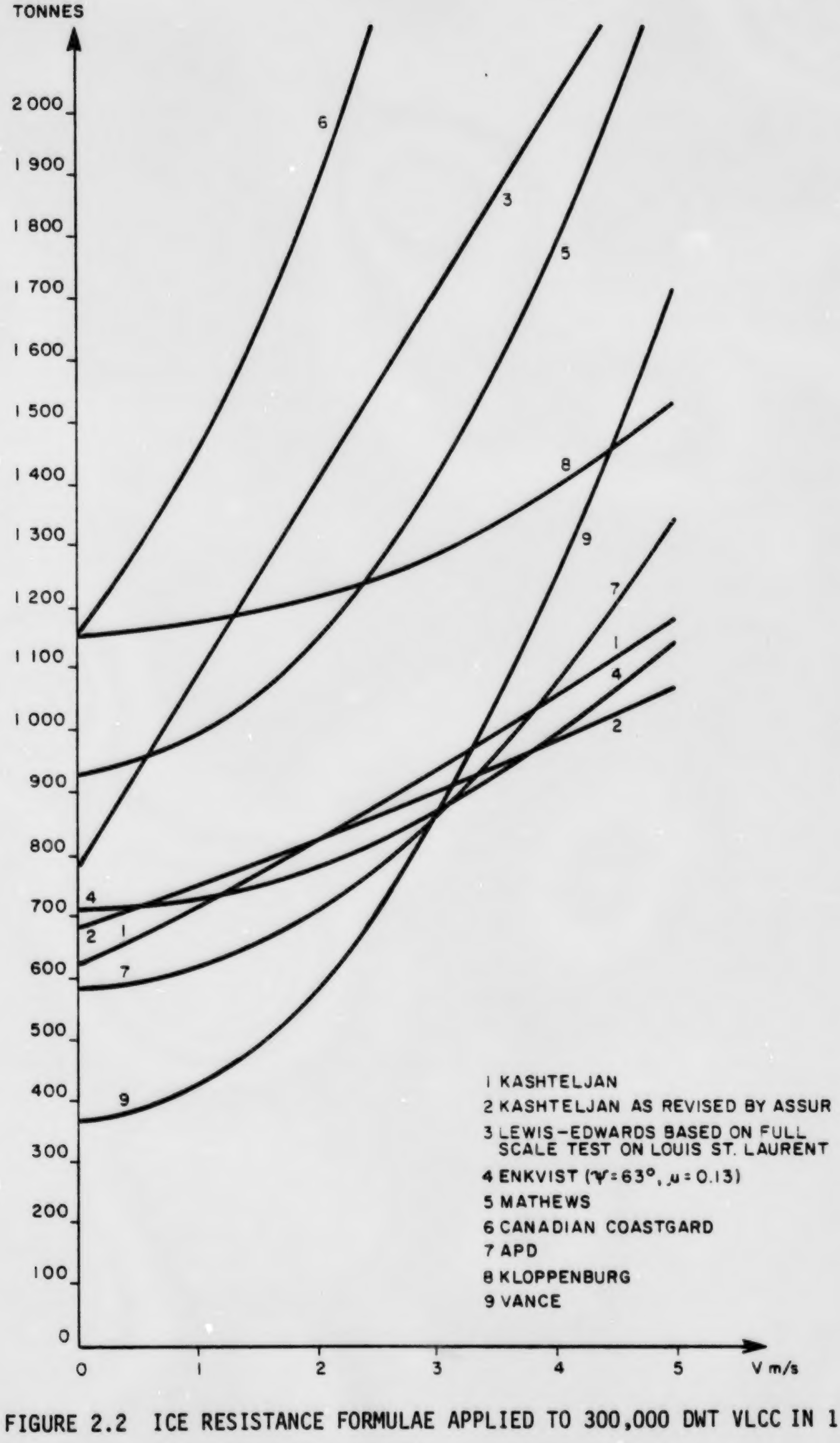
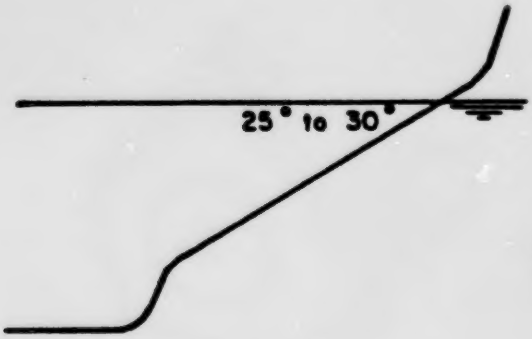
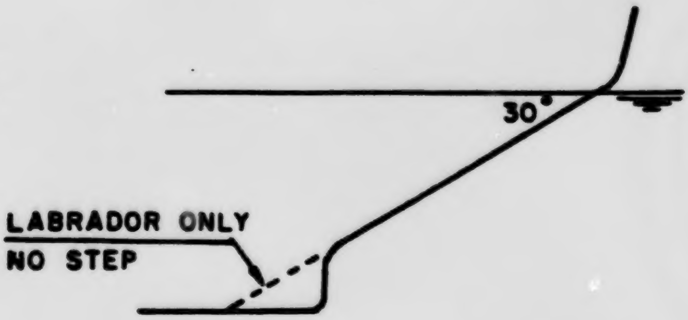


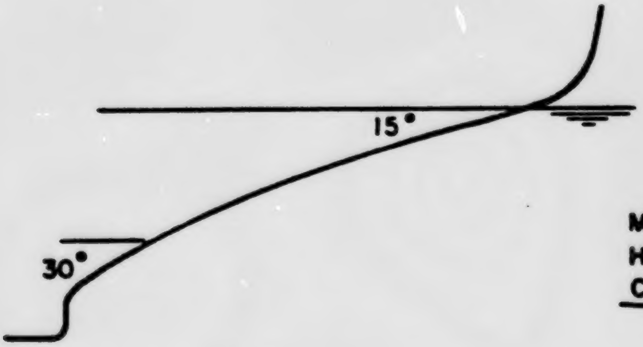
FIGURE 2.2 ICE RESISTANCE FORMULAE APPLIED TO 300,000 DWT VLCC IN 1.5 m ICE



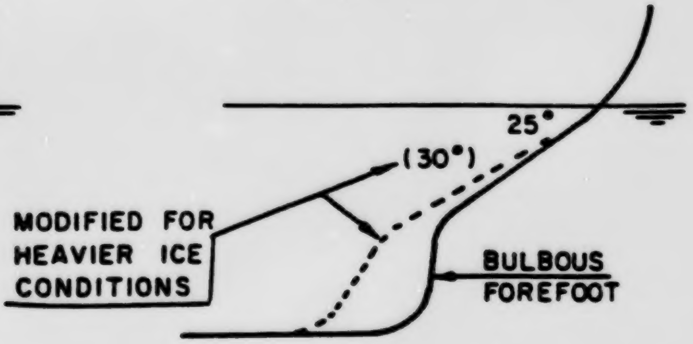
1. FINNISH/RUSSIAN TYPE
MOSKVA CLASS



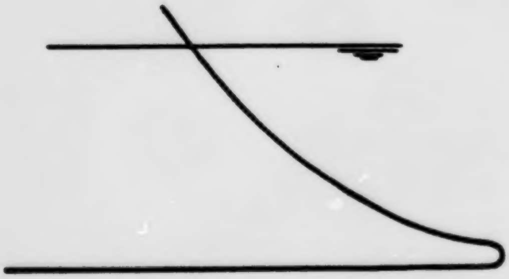
2. CANADIAN TYPE
LOUIS S. ST. LAURENT



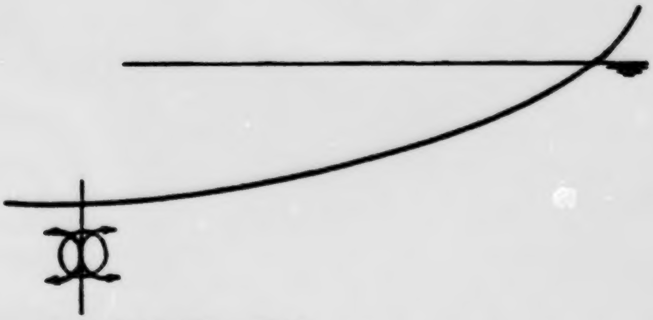
3. M.I.T. TYPE
MANHATTAN



4. IMPERIAL ACADIA
IMPERIAL BEDFORD
ICE CLASS I TANKERS FOR
ST. LAWRENCE GULF SERVICE



6. JESSEL / SCOT ALEXANDER TYPE



5. HAMBURG TUG

FIGURE 2.3 TYPES OF ICE BOW (TO 1976)

3.0 DATA BANK, SCANNING PROCEDURE AND DATA EXTRACTION

3.1 Scanning Procedure

During the three operating seasons of the M.V. Arctic in which performance data were collected 163 data tapes were recorded, each with a maximum capacity of approximately 5 million data samples. Because of the enormity of this data collection it was essential to determine during the scanning activity which of the tapes contained information most likely to be useful in this project. The three basic criteria were the ship transit mode, the presence of performance parameters and the availability of documentation. Other information required included the details on the data recording format and the data conversion equations. With this in mind, a review of all the available documentation on the sea trials and the data collection system was initiated.

The starting point chosen was a study of the Master Data Package. This document contains a summary of the contents of each of the 163 tapes, as well as a description of the data collection system and the status of the sensors and their conversion equations during each of the voyages. From this examination it was determined that the data collection system installed on the M.V. Arctic included 212 sensors, 39 of which recorded parameters relevant to the ship's performance, and therefore this project. The examination also revealed the unfortunate fact that many of the performance parameters were frequently not recorded due to sensor difficulties. The most critical parameters, thrust, torque, shaft speed and ship speed were absent from most of the 1979 tapes and, in 1980, ship speed and shaft speed were still not available. Only in 1981 were these items properly recorded. To illustrate this, a three year summary of the sensor status for the major performance parameters is shown in Table 3.1. Given backup records in the form of manual logs, some of the data from 1979 and 1980 could possibly be used; however, they proved to be available only for the dedicated trials.

TABLE 3.1

Sensor Status Summary

Year		1979							1980							1981								
Voyage		1	2	3	4	5	6	7	1	2	4	5	6	T1	T2*	1	2	6	7	8	9	10	T1	T2**
Measured Parameter	Channel No.																							
Shaft Thrust	198								●		●	●	●	●	●	●	●	●	●		●	●	●	●
Shaft Torque	208					●	●	●	●		●	●	●	●	●	●	●	●	●		●	●	●	●
Shaft RPM	242											○	○		●									
Shaft RPM	211															●	●	●	●		●	●	●	●
Engine RPM	241	○																						
Propeller Pitch	201	○		●		●	●	●	●		●	●	●	●	●	●	●	●	●		●	●	●	●
Ship Speed	236	○						●						●	●	●	●	●	●		●	●	●	●
Wind Speed	233					●	●	●	●		●	●	●	●	●	●	●	●	●		●	●	●	●
Wind Direction	234	○				●	●	●	●		●	●	●	●	●	●	●	●	●		●	●	●	●
Forward Accel.	1,2,31	●	●	●	●			●	●	●	●	●	●	●	●	●	●	●	●	○	●	●	●	●
Aft. Accel.	212,213, 214								●		●	●	●		●	●	●	●	●	○	●	●	●	●

*
T1 - Saguenay River Trial
T2 - Baffin Island Trials

**
T1 - Lake Melville Trials
T2 - Admiralty Inlet Trials

● Data Available
○ Data Partially Available

These facts prompted the decision to spend the most effort on the 1981 voyages and the dedicated trials data tapes. Some tapes from the 1979 and 1980 seasons recorded during heavy ice conditions were also to be examined.

Included in the information provided by the Master Data Package on each of the data tapes is a description of the transit mode of the ship. Based on these descriptions, the tapes which contained any data recorded during continuous mode transit through broken, old, new or open water channels were selected. Approximately 46 tapes from the 1981 Early and Late seasons and the Lake Melville dedicated trials, along with 20 more tapes from the 1980 Saguenay River and Baffin Island trials and the 1979/80 voyages were accumulated.

For the purpose of analyzing the continuous mode ice-transiting performance of the ship, the following parameters were chosen for extraction from the data tapes:

- Thrust
- Torque
- Ship Speed
- Shaft RPM
- Wind Speed
- Wind Direction
- Propeller Pitch

Although other data such as ice cover, bubbler status and draught were supposed to be recorded on tape, it was felt that they were best taken from the manual logs. This proved to be the right decision as later spot checks indicated these channels were unreliable.

In order to convert the recorded data into engineering units, the proper calibration equations also had to be identified. For this, the special reports on the regular voyages and dedicated trials for each year were consulted, as well as the Master Data

Package. It became apparent that there were major discrepancies between the two in the areas of channel assignment and conversion equations; however, a careful study of the matter indicated that the equations from the special reports were most likely the correct ones. Table 3.2 contains a list of the equations that were used for the selected parameters in the subsequent data extraction process.

3.2 Data Extraction

Although the data collection system onboard the M.V. Arctic used a standard 9-track tape drive for data storage, the recording format was non-standard, to permit more efficient usage of the tape. This means that the tapes can be read with any computer system incorporating a 9-track tape drive, provided that special software is written to control the drive and access the records.

The equipment used by GasTOPS Ltd. to read and analyze the data tapes includes a DEC PDP-11 mini-computer with a tape drive, hard disk drive, video graphics terminal and matrix printer. Some software was supplied by the contractor, but it was found that it would require extensive modification to be made suitable for use on the above equipment. Rather than do this, it was decided to modify another available program which was originally designed as a general purpose utility for reading tapes with foreign file formats. This program already incorporated the code required to control the tape drive and transfer data in block form from tape to computer memory. The code required to interpret the data blocks, convert the measured parameters into engineering units and present the data in an easy to use form had to be added.

Each tape is comprised of one or more files which contain a collection of the data recorded during a particular time period. The files can extend for the entire length of the tape,

TABLE 3.2 DATA CONVERSION EQUATIONS

Parameters	Chan.	Units	EQUATIONS					
			1979 Season	1980 Saguenay River	1980 Season	1980 Baffin Island	1981 Season	1981 Lake Melville
THRUST	198	kN	(2.4 x Data) - 1155.	(2.4 x Data) - 1155.	(2.3 x Data) - 1176.8	(2.299 x Data) - 1176.8	(2.299 x Data) - 1176.8	(2.299 x Data) - 1176.8
TORQUE	208	kNm	(0.92 x Data) - 320.1	(0.92 x Data) - 320.1	(0.887 x Data) - 606.9 (1.774 x Data) - 181.7 *	(1.774 x Data) - 1121.1*	1.801 x (Data - 102.4)	1.801 x (Data - 102.4)
SHAFT SPEED	211	RPM	-	-	-	-	61440./(Data + 18.4)	61314./(Data - 4.58)
SHIP SPEED	236	Knots	(0.0196 x Data) +).86	Data x 0.1 (1 - Data/1024) - 1.18	0.0293 x Data	0.0293 x Data	0.0293 x Data	0.0293 x Data
PROP PITCH	201	Deg	(0.0195 x Data) - 10.	(0.0195 x Data) - 10.	0.0195 x Data) - 10.	0.0371 x Data) - 15.	(0.0371 x Data) - 15.	(0.0371 x Data) - 15.
WIND SPEED	233	Knots	(0.136 x Data) - 69.5	(0.136 x Data) - 69.5	(0.136 x Data) - 69.5	(0.136 x Data - 69.5	(0.136 x Data) - 69.5	(0.136 x Data) - 69.5
WIND DIRECT.	234	Deg	(-0.352 x Data) + 360.	(-0.352 x Data) + 360.	(-0.352 x Data) - 360.	(-0.352 x Data) + 360.	(-0.352 x Data) + 360.	(0.352 x Data) +360.
ACCEL.	All	m/s/s	(0.048 x Data) - 24.5	(0.048 x Data) -24.5	(0.048 x Data) - 24.5	(0.048 x Data) - 24.5	(0.048 x Data) - 24.5	(0.048 x Data) - 24.5
					* Voyage 6 only	* 4 other equations required for the period October 27 to October 31 (see report TP3202)		

as in the case of a voyage recording; or they may extend only a few minutes, as in the case of a single test run during the dedicated trials. Each file consists of a header block followed by a variable number of data blocks. The header block contains the starting date and time plus the recording mode, sampling rate and minimum and maximum alarm limits for each sensor. Each data block contains the current date and time plus a variable number of data word pairs, where the first word specifies the sensor address and the second word contains its measured value.

The first program modifications made were to recognize and decipher the header blocks and then extract and dump the following data blocks on a printer for examination. When the correct operation of this code was proven, further additions were made to scan all the data blocks, ignore data from unwanted sensors and store the desired performance parameter data in a disk file. The disk files utilized a more convenient format and allowed very fast data access compared to the original tape files.

In order to minimize the size of the new disk files, an averaging process was also included wherein the data for all channels are reduced to two samples per second regardless of the original recorded sampling rate. For the purposes of this project, this averaging process was judged to be acceptable and even desirable, since it tended to reduce the data scatter.

Before any method for analyzing the data could be established, it was felt that the recorded signals would have to be examined to determine their accuracy, noise levels and general characteristics. To this end, a second program was written to display any chosen parameter graphically on the video terminal. This program operates by reading data from the disk file previously created and then plotting one parameter versus another for some specified time segment. This visual technique proved to be a very effective way of examining the various data signals, and it became clear that it is the only way steady state data points can be reliably selected. A computer program which automatically scanned the data for steady state segments and then selected an appropriately averaged value would be too complex and uncertain.

As a result of these observations, the data display program was refined so that it automatically scans a disk file for a specified parameter and produces a complete time history of that item by plotting it in segments covering 15 minute periods. Other features added were the capability to repeat this process for any number of specified parameters and to provide a copy of each plot on the matrix printer Sample plots for a group of performance parameters extracted by this program are shown in Appendix B.

With these two programs in place, a production procedure was set up to analyze all the tapes of interest. First, the tape reader program was used to perform the initial extraction and reformatting of the relevant data from each of the tapes. This process was the most time consuming due to the very high density of the recorded data and the slow speed of the tape drive. In addition to this, a number of difficulties were encountered due to flaws in some of the tapes. Each new problem required program modifications to circumvent it and also necessitated re-reading of many tapes. The problems encountered include:

- bad file header blocks
- bad data blocks
- bad tape segments resulting in hardware read errors
- missing end-of-tape marks causing the tape to run off the reel.

Most of the tapes encountered which have one or more problems are listed in Table 3.3.

Once the data of interest was stored on disk, the second phase involving the production of time history plots proceeded

TABLE 3.3

Defective Tapes

Tapes With No End-of-Tape Marks

80-6-21
80-T2-8
81-7-7
81-8-11
81-9-14
81-9-15
81-10-38
81-10-42B
81-10-47

Tape File With Bad Header Blocks

80-T1-5.1
80-T1-7.1
81-9-12.4
81-9-12.11
81-9-12.12
81-T1-10.1

Tape Files With Unreadable or Bad Data Blocks

79-7-37.1
80-T1-1.1
80-T1-2.1
80-T1-8.1
80-T2-3.2
80-T2-3.3
80-T2-3.7
80-T2-9.2
80-T2-18.1
81-6-2.7
81-7-7.3
81-8-11.2
81-9-13.8
81-9-14.3
81-9-15.5
81-9-17.1
81-9-20.2

quicker. The hardcopy plotting was very slow, but this was alleviated by setting up the program so that it could be run continuously overnight. This procedure worked well, except for one recurring problem which was a program abort which was caused by bad time recordings. The time signal is critical to the searching and plotting functions of the program. The bad recordings appeared in numerous forms making it virtually impossible to formulate a software solution which handled all the occurrences.

During the course of the project all of the relevant tapes from the 1981 voyages and the Lake Melville, Baffin Island and Saguenay River dedicated trials, plus the four tapes* from the 1979 and 1980 voyages, were processed in this manner.

Seven performance parameters were included, resulting in approximately 3,000 plots covering about 100 hours of data. To permit analysis of the data contained in these plots, selected points must be scaled off manually. The selection process requires that the manually recorded logs be consulted first; to determine the exact time when a special test run was made, or when other interesting conditions occurred. Given an exact date and time, data points can be pinpointed on the appropriate plots. This exercise is the final step in reducing hours of recorded data to a handful of data sets which can be readily analyzed. The discussion on this phase of the project is presented in the following chapters.

---

* These four tapes were 79-7-37, 80-5-15, 80-5-17 and 80-6-21. They were selected because they were the only ones which recorded transit through ice and also had measurements of R.P.M.

4.0 REDUCTION AND SUMMARY OF DATA

4.1 Introduction

The sets of data summarized in this Section are discussed under the following main headings:

- 1981 Dedicated Trials, Lake Melville
- 1980 Dedicated Trials, Baffin Island
- 1979/80 Dedicated Trials, Saguenay River
- Voyage Data

Emphasis has had to be placed on the dedicated trials because insufficient data on ice conditions were recorded during the voyages. However, some selected voyage data have been used to give a general assessment of the M.V. Arctic's performance in ice floes and broken channels.

The General procedure for compiling and validating the essential data is described in Subsection 4.2 covering the Lake Melville trials from which the first set of performance data was reduced and summarized. The procedure for summarizing the data from the Baffin Island and Saguenay River trials was similar.

In the case of the Lake Melville trials and the Baffin Island trials, the Doppler tape speeds were initially taken as being correct and the ice resistance for each run was calculated on this assumption. It was subsequently decided, however, that the manually measured speeds were more accurate except for a few apparently inconsistent results which it was felt might have an error of a whole minute in the stop-watch timing. The inconsistent results were therefore omitted and the remaining manually measured speeds were used to derive an equation for calibrating the Doppler tape speeds. New values of ice resistance were then calculated to correspond with the modified speeds.

In the case of the Saguenay River trials the speeds were not checked by manual measurement and no calibration of Doppler tape speeds was possible.

4.2 1981 Dedicated Trials, Lake Melville

4.2.1 Measured Data

These trials were carried out between 27 February and 3 March, 1981. The data tapes were used to produce time based plots of the items in columns 7 to 11 and 14, 15 of Table 4.1. This computer work was carried out as described in Section 3 and the conversion equations given in Table 3.2 were used. For shaft torque and pitch response these equations are different from those given in the Master Data Package but agree with the equations given in Report TP 3632 (vol. 1), 1981 Early Season. The differences apparently reflect updating of the system.

Column 7 of Table 4.1, ship speed, is as read directly for the data tape plots for each run which reflect the signals direct from the Doppler Log. Various difficulties reported with the Doppler Log in the trials could perhaps reflect problems of the log repeater used in the manual notes. Comparisons of the tape Doppler data with often conflicting data derived from the log repeater, on-ice markers, T.V. recordings and the various comments made on these last three methods show the Doppler tape data are consistent in themselves but not necessarily accurate. Subsequent analyses show that these data require calibration, using the manually measured speeds as described in Subsection 4.2.4.

Column 8 of Table 4.1 represents shaft speed directly as lifted from the tapes' plots.

It should be mentioned that the manually recorded times for starting and stopping each run, columns 5 and 6, are fast compared with the actual pattern of events by about 6 minutes from the tape times governed by the computer clock. Runs 1 and 3 from the evidence of the tape records show that the controls were changed during each of these runs and hence four runs 1a, 1b, 3a, 3b are obtained during those intervals of time.

The ice thicknesses, column 12, Table 4.1, were obtained by noting the site numbers for the time of day of each run from the charts supplied in Vol. 2 of Report TP 3646E and then reading ice thicknesses from Table 7.4 of that report. Where ice thickness was measured at the beginning and the end of a run, an average value was taken. For most of the runs this gives ice thicknesses slightly different from those noted in Table 7.4 of Vol. 1 of the above report. The snow cover was apparently very small or non-existent during the trials. The average flexural ice strength in the trials vicinity was 267 kPa and the ice Modulous of Elasticity was 2322 MPa as given in Report NRC LTR-SH-301.

4.2.2 Model Correlation

Values of propeller pitch response, propeller thrust and torque were taken off the tape plots and comparisons with the 1:30 self propelled model overload data in clear water (N.R.C. Report LTR-SH-349) were made on the basis of both thrust and torque coefficients, K_T and K_Q. The mean model pitch from these methods and the corresponding values of the K_T and K_Q at this mean pitch were obtained. These values of K_T and K_Q were used to derive the correlation T_A and Q_A (and P_{SA}). It was found that the model correlation thrust, T_A was within 1.5%, on an average, of the measured values. No difference between measured ship and correlated model for T and Q was more than about 4%.

The corresponding model correlation propeller pitch obtained from this mean of K_T identity was on an average 3.5 degrees higher than that from the tape data using the equation of Report TP 3637. These higher correlation pitch values are closer to the propeller design values and it is considered accurately reflect the actual operating conditions. The correlation factors for thrust, torque and pitch for each run are shown on Figure 4.1.

Due to this good correlation it is considered that, for these propulsion items, the self propelled model clear water overload data reflect the ship performance, at these ice breaking speeds, with a high degree of precision and that these model data may be used with confidence to represent the ship performance. The correlated values of propeller pitch, thrust and shaft power are given in columns 9, 10 and 11 of Table 4.1. These data are plotted in Figures 4.2, 4.3 and 4.4 where each point corresponds to the measured ship speed. Average lines are drawn for each parameter.

4.2.3 Derivation of Ice Resistance

Table 4.2 repeats the ice thicknesses, ship speeds and propeller thrusts in columns 3, 4 and 5 and adds the duct thrust, T_D, total thrust T and pull, over and above thrust deduction and ship open water resistance, available to overcome ice resistance in columns 6, 7 and 8. These latter items are also plotted on Figure 4.4 The open water resistance is shown in Figure 4.5.

4.2.4 Modification of Data

Discussion with Melville Shipping Ltd. subsequent to the reduction of the data discussed above and summarized in Table 4.1 and 4.2 cast doubt on the accuracy of the speeds obtained directly from the Doppler tape data. It was finally concluded that the manually measured speeds were the more accurate although there were some major discrepancies which suggested possible errors of one minute in the stop-watch timing of some of the speeds.

The procedure adopted, therefore, was to calibrate the Doppler tape data with the manually measured speeds after omission of runs showing major discrepancies. On this basis, the equation derived for calibration of the Lake Melville speed data was:

$$V = 0.772 + 0.943\ V_D \text{ knots}$$

where V_D is the speed from the Doppler tape data.

The resulting modified speeds and corresponding modified values of R_i are shown in Table 4.3.

4.3 1980 Dedicated Trials, Baffin Island

These trials were conducted from October 25 to 31, 1980, during which period 41 tests were performed as the ship travelled from Pond Inlet to Nanasivik. Descriptive details are given in Report No. 2P 3646 E.

Ice thickness was measured at regular intervals and measurements of ice properties were made in each test area. Average properties were as shown in Table 4.4.

TABLE 4.4

BAFFIN ISLAND TRIALS - AVERAGE ICE PROPERTIES

Test Site	Thickness h, m	Strength σ_f, kPa	Modulus E, MPa	$\frac{E}{\sigma_f}$	Ice Temp. °C	Brine Volume
2*	0.185	235 ± 25	400 ± 35	1700	-2.8	0.126
2**	0.185	350 ± 80	525 ± 75	1500	-2.8	0.126
3	0.315	340 ± 85	1090 ± 390	3200	-3.8	0.092
4	0.39	330 ± 115	1320	4000	-4.7	0.070
5	0.32	410 ± 70	1400 ± 250	3400	-4.9	0.061

* beam length parallel to C-axis

** beam length perpendicular to C-axis

The data extracted from the Baffin Island trials with bubbler off is summarized in Table 4.5 for speeds taken from the Doppler tape. These speeds were then compared with the manually measured speeds and modified according to the following calibration equations:

Runs 1.1 to 23, $V = 0.924 + 1.03\ V_D$ Knots

Runs 24 to 35.2, $V = 0.0876 + 1.02\ V_D$ Knots

The modified speeds and corresponding modified values of R_i are shown in Table 4.6.

The data from the Baffin Island trials with bubbler on is summarized in Table 4.7 after modifying speeds and values of R_i as previously described.

4.4 1979/80 Dedicated Trials Saguenay River

The Saguenay River Trials were carried out during January 18 to 21, 1980 near Port Alfred, Quebec. Details are described in Report No. TP 2627, where Appendices 2 and 3 give the mechanical ice properties measured and the general characteristics of the ice.

Ice samples were obtained by cutting pieces of ice with a chain saw from ice blocks broken and pushed up along the side of the ship. The average salinity of the ice was 1.3 o/oo and the temperature varied from -15.5° C at the surface to -5.9° C at a depth of 35 cm.

No direct measurements of flexural ice strength were made by testing cantilever beams and the flexural strength was computed in Appendix 2 of TP 2627 by Frankenstein's formula (1970)

$$V = S \left(\frac{-49.185}{T} + 0.532 \right) \times 10^{-3} \qquad (1)$$

where V = brine volume
S = salinity in parts per thousand
T = temperature in ° C

and the formula of Weeks and Assur (1967)

$$\sigma_f = \sigma_{f\omega} \; (1 - 2\sqrt{V} + V) \qquad (2)$$

where $\sigma_{f\omega}$ is the flexural strength of fresh water ice. Substituting 1.3 and -8° C for S and T in equation (1) gives $V = 0.00868$ and $\sqrt{V} = 0.093$. Unfortunately the value of $\sqrt{V}$ was used for V in equation (2) by the author of Appendix 2 of TP 2627 who derived a value for σ_f of 353 kPa instead of 600 kPa, assuming σ_{fw}= 730 kPa. Appendix 2 of TP 2627 mentions tests by Ohstrom and Den Hartog (1976) on cantilever beams in fresh water giving average values of $\sigma_{f\omega}$= 445 kPa. Taking 445 kPa as a low value and 730 kPa as a high value the strength of the brackish ice of the Saguenay River might have been anything from 366 to 600 kPa. Mathews proposes 450 kPa as being a reasonable value (see Appendix C) but, as pointed out in Section 5.2 the comparison of low speed data (Figure 5.7) indicates a slightly smaller strength closer to that of the Baffin Bay trials.

No manually measured checks of speed were made during the Saguenay River trials and the Doppler tape speeds have been adopted. It is emphasized, however, that manual measurements during both the Lake Melville and Baffins Bay trials indicated that the Doppler tape speeds were generally too low and this must be borne in mind when analyzing the Saguenay River data which is summarized in Table 4.8.

4.5 Voyage Data

As explained in Section 3, the completeness and quality of data available from the voyages is generally poor. A preliminary assessment of the availability of useful data was made by review of the master data package and selected video tapes. From this review and trials reports, voyage segments which appeared potentially assessable to analysis of performance in pack ice or broken channels were identified. Several other interesting periods of broken ice operations were observed on video during the dedicated trial periods, but these were presumably between tests and were without any documentation in the trial logs.

In general, the video tapes are not of sufficient quality to extract useful details of broken ice conditions. There is no scale, and the 10 times normal speed makes viewing very difficult. Attempts to slow down or freeze the video produced a blurred indistinct image.

No specific written description was found to the condition of a broken ice channel in any of the MV Arctic records or contractor reports.

Tables 4.9 to 4.13 show some 1981 early season data from Lancaster Sound, the approach to Nanisivik, Admiralty Inlet and off Cape Farewell, including some travel in continuous level ice. These tables are examples from about fifty sheets of manuscript tabulations of voyage data from which data were selected for analysis of performance in pack ice and broken channels. The selected data are summarized in Table 4.14.

TABLE 4.1

LAKE MELVILLE TRIALS DATA - LEVEL ICE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Col.
Db.	File	Run	Bub.	Stt.	Stp.	V_D m/s	Ns RPM	P DEG.	Tp KN	Ps KW	h m	Snow CM	Wind S Kn.	Wind Dir. DEG.	
27 February '81	MM 8109.001 81-T1-8 C8	1a		11:08		.540	123	14.25	349	3319	.64		5.4	58	
		1b			11:16	.797	128	15.10	418	4071	.64		9.8	58	
		2		11:16	11:20	1.157	132	16.20	505	4976	.58		10.9	54	
		3a		11:20		1.878	138	17.20	607	6440	.64		13.0	55	
		3b			11:30	2.444	140	19.03	715	8223	.64		15.0	55	
		4		11:30	11:46	2.984	144	20.15	821	10143	.68		14.0	59	
		5	On	11:46	11:49	1.080	136	17.13	610	6256	.59		7.9	59	
		6	On	11:49	12:00	1.749	140	18.80	732	8156	.67		10.0	54	
		7	On	12:00	12:10	2.006	144	20.99	912	11230	.77		9.0	52	
	MM 8108.do1 81-T1-9 C9	8	On	12:45	12:50	.823	136	17:55	630	6592	.83		0.4	85	
		9	On	12:50	12:63	1.646	140	18.93	741	8322	.69		2.3	77	
		10	On	12:63	13:09	1.903	144	20.68	998	11049	.80		5.0	83	
		11		13:09	13:17	.360	136	17.50	642	6653	.75		0.3	76	
		12		13:17	13:35	.797	140	18.90	754	8522	.86		0.2	75	
1 March '81	MM 8105.001 81-T1-11 C11	13	On	09:24	09:39	.411	140	18:85	767	8522	.93	7	12.5	245	
		14	On	09:31	10:00	1.132	144	20:55	912	11013	.86	7.8	5.5	252	
		15		10:00	10:07	1.029	144	20.68	921	11157	1.07	8.5	1.5	142	
		16		10:07	10:17	.668	142.5	19.65	835	9724	.92	8	4.0	136	
		17		10:17	10:30	1.749	144	20.75	903	11121	.86		5.5	131	
	*	18	On	12:11	12:20	1.595	142	20.90	925	11411	.80		2.0	174	

* File Number MM 8104.001 (81-T1-12, C12)
Beam Test Results:

V_D = Doppler Tape Speed

Average flexural strength σ = 267 kPa
Modulus of Elasticity E = 2322 MPa

TABLE 4.2

LAKE MELVILLE TRIALS - LEVEL ICE

DERIVED TOTAL THRUST AND ICE RESISTANCE BASED ON DOPPLER TAPE SPEEDS

1	2	3	4	5	6	7	8	Col.
Run	Bub.	h m	V_D m/s	Tp kN	TD KN	T KN	RI KN	
1a		.64	.5402	349	283	632	532	
1b		.64	.7974	418	324	742	619	
2		.58	1.157	505	375	880	728	
3a		.64	1.878	607	414	1021	804	
3b		.64	2.444	715	465	1180	913	
4		.68	2.984	821	501	1322	1012	
5	On	.59	1.080	610	451	1061	895	
6	On	.67	1.749	732	517	1249	1034	
7	On	.77	2.006	912	647	1559	1299	
8	On	.83	.8231	630	520	1150	1016	
9	On	.69	1.646	741	539	1280	1077	
10	On	.80	1.903	898	647	1545	1308	
11		.75	.3601	642	549	1191	1057	
12		.86	.7974	754	638	1392	1250	
13	On	.95	.4116	767	668	1435	1293	
14	On	.86	1.132	912	720	1632	1459	
15		1.07	1.029	921	738	1659	1504	
16		.92	.6686	835	701	1536	1393	
17		.86	1.749	903	670	1573	1349	
18	On	.80	1.595	925	688	1613	1413	

TABLE 4.3

LAKE MELVILLE TRIALS - LEVEL ICE

MODIFIED DOPPLER TAPE SPEEDS
AND CORRESPONDING RESISTANCE

1	2	3	4	5	6	7	8
		Doppler Data			Modified Data		
Run	h m	Knots	m/s	R_i	Knots	m/s	R_i
1a	0.64	1.05	0.54	532	1.76	0.91	512
1b	0.64	1.55	0.80	619	2.23	1.15	544
2	0.58	2.25	1.16	728	2.89	1.49	703
3a	0.64	3.65	1.88	804	4.21	2.17	764
3b	0.64	4.75	2.45	913	5.25	2.70	888
4	0.68	5.79	2.98	1012	6.23	3.21	977
5	0.59	2.10	1.08	895	2.75	1.42	865
6	0.67	3.40	1.75	1034	3.98	2.05	1004
7	0.77	3.90	2.01	1299	4.45	2.29	1254
8	0.83	1.60	0.82	1016	2.28	1.17	971
9	0.69	3.20	1.65	1077	3.79	1.95	1042
10	0.80	3.70	1.91	1308	4.26	2.19	1263
11	0.75	0.70	0.36	1057	1.43	0.74	1012
12	0.86	1.55	0.80	1250	2.23	1.15	1205
13	0.95	0.80	0.41	1293	1.53	0.79	1243
14	0.86	2.20	1.13	1459	2.85	1.47	1424
15	1.07	2.00	1.03	1504	2.66	1.37	1459
16	0.92	1.30	0.67	1393	2.00	1.03	1348
17	0.86	3.40	1.75	1349	3.98	2.05	1309
18	0.80	3.10	1.60	1413	3.70	1.90	1368

TABLE 4.5

1980 BAFFIN ISLAND LEVEL ICE - TRIALS DATA

BUBBLER OFF

1	2	3	††APP.D 4	5	6	7	8	9	10	†† 11
File MB80	Date 1980	Area	Ice Test Site	Run	T Kn.	Q KN.m	V Kn.	P Deg.	R KN	h m
01.001	26/10	NBI	SITES 2 AND 3	1.1	170	115	2.4	7.3	220	.183
01.002				1.2	310	240	5.7	12.4	335	.192
02.002				1.4	700	665	11.7	20.8	500	.186
03.002				3.3	325	235	6.5	12.7	295	.195
				3.4	175	107	2.95	7.8	190	.224
05.004	27/10			6.1	180	110	1.2	8.0	255	.312
05.005				6.2	335	242	4.5	12.6	395	.325
05.006				6.3	615	485	8.35	16.4	585	.307
05.012				10.1	171	96	1.6	7.9	230	.300
05.016				10.3	460	336	7.2	14.4	430	.300
05.014	28/10			10.4	800	687	10.1	20.8	715	.300
08.004	29/10	LS	4	15.0	800	693	10.93	20.8	650	.335
10.004			4	24.0	397	300	7.12	15.0	360	.350
11.05			4	27.0	320	210	2.8	11.2	460	.357
17.001	31/10	STR	5	30.1	183	128	2.3	10.4	260	.330
17.002			5	31.1	360	280	6.4	14.0	365	.330
17.002			5	31.2	360	280	6.8	14.0	340	.330
17.002			5	32.2	450	373	7.8	16.4	420	.345
18.001			5	33.2	775	707	12.3	20.8	510	.330
18.001			5	34.2	497	380	8.5	16.2	400	.330
18.001			5	35.2	240	160	3.75	8.3	285	.310

†† Report TP3 202

V_D = Speeds from Doppler Log data

TABLE 4.6

BAFFIN ISLAND TRIALS - LEVEL ICE
BUBBLER OFF
MODIFIED DOPPLER TAPE SPEEDS
AND CORRESPONDING RESISTANCE

1	2	3	4	5	6	7	8
		Doppler Data			Modified Data		
Run	h m	Knots	m/s	R_i kN	Knots	m/s	R_i kN
1.1	0.183	2.4	1.24	220	3.40	1.75	185
1.2	0.192	5.7	2.93	335	6.80	3.50	290
1.4	0.186	11.7	6.02	500	12.98	6.68	390
3.3	0.195	6.5	3.35	295	7.62	3.92	245
3.4	0.224	2.95	1.52	190	3.96	2.04	145
6.1	0.312	1.2	0.62	255	2.16	1.11	225
6.2	0.325	4.5	2.32	395	5.56	2.86	350
6.3	0.307	8.35	4.30	585	9.52	4.90	500
10.1	0.300	1.6	0.82	230	2.57	1.32	205
10.3	0.300	7.2	3.71	430	8.34	4.29	370
10.4	0.300	10.1	5.20	715	11.33	5.83	620
15.0	0.335	10.93	5.63	650	12.18	6.27	540
24.0	0.350	7.12	3.67	360	7.36	3.79	350
27.0	0.357	2.8	1.44	460	2.95	1.52	455
30.1	0.330	2.3	1.18	260	2.44	1.26	260
31.1	0.330	6.4	3.29	365	6.62	3.41	360
31.2	0.330	6.8	3.50	340	7.03	3.62	325
32.2	0.345	7.8	4.02	420	8.05	4.14	400
33.2	0.330	12.3	6.33	510	12.65	6.51	480
34.2	0.330	8.5	4.38	400	8.77	4.51	380
35.2	0.310	3.75	1.93	285	3.92	2.02	280

TABLE 4.7

BAFFIN ISLAND TRIALS - LEVEL ICE TRIALS DATA

BUBBLER ON

1	2	3	4	5	6	7	8	9	10	11
File M380	Date 1980	Site	Run	T.kN	Q.kN	V_D.m/s	V_M.m/s	P.deg.	R_i.kN	h. m.
05010	27/10	3	9.3	320	212	2.88	3.44	12.8	268	0.29
05011	27/10	3	9.4	170	90	1.03	1.54	7.8	195	0.29
10003	29/10	4	22.0	270	192	1.29	1.72	11.8	332	0.44
10003	29/10	4	23.0	270	192	1.13	1.72	11.8	332	0.44
17002	31/10	5	30.2	182	125	1.29	1.36	10.4	227	0.33
17002	31/10	5	32.1	450	375	4.12	4.25	16.4	376	0.34
18001	31/10	5	33.1	770	705	6.28	6.45	20.8	503	0.33
18001	31/10	5	34.1	485	378	4.42	4.56	16.2	386	0.33
18001	31/10	5	35.1	240	160	2.01	2.09	8.3	255	0.33

NOTES:
V_D = Speeds from Doppler data.
V_M = Speeds modified by calibration with manually measured speeds.
R_i = Ice resistance corresponding to V_M.
Site 3 is Navy Board Inlet.
Site 4 is Lancaster Sound.
Site 5 is Strathcona Sound.

TABLE 4.8

1980 SAGUENAY RIVER TRIALS DATA - LEVEL ICE

Manually Recorded TP 2627						From Tape Recordings							Derived	
Run	Date 1980	h m	Snow CVR cm	N RPM	BUBBLR.	File MS80	Tp kN	Q kN.m	V_D m/s	p +++	Wind Speed Kn	Wind Dirn. Deg.	p Deg.	Ri kN
1-DS	20/1	.4445	2.5			01.001	100	70		.4	16	240		
1-S(¼)				114.4			160	100	0.56	1.8	"	"	7.3	230
1-½				127.1			280	180	0.93	3.2	"	"	10.05	415
1-3/4				127.1			360	275	2.06	4.4	"	"	13.7	480
1-F				133.5			595	525	3.91	5.8	"	"	19.7	660
2-¼	20/1	.4445	2.5	127.1		01.001	160	100		1.8	23	240		
2-½				127.1			280	180	0.82+	3.0	"	"	10.1	420
2-3/4				127.1			360	280	1.80+	4.2	"	"	13.7	505
2-F				133.5		02.001	580	525	3.66	5.8	17	240	19.7	660
4-S(¼)	21/1	.4842	3.75	127.1		06.001	140	107		1.2	29	90		
4-½				127.1			175	120	0.69	1.6	"	"	7.0	270
4-3/4				127.1			260	167		3.0	"	"		
4-X				127.1			340	260	1.03	4.0	"	"	12.7	540
4-F				133.5			570	525	3.12	5.7	"	"	19.2	730
6-S	21/1	.5017	3.75	114.4	ON	07.001	100	75		.4	18	80		
6-½				114.4	ON		160	110		1.8	"	"		
6-3/4				127.1	ON		280	187	0.62+	2.9	"	"	10.1	440
6-X				127.1	ON		380	305	1.39	4.4	22	85	14.7	585
6-F				133.5	ON		640	540	3.04	5.8	30	80	20.7	815
7-½	21/1	.3715‡	3.75	114.4		08.002	160	105	0.26	1.8	18	30	5.7	260
7-3/4				130.3			260	165	0.41	2.8	"	"	8.2	390
7-X				127.1			340	255	1.29	4.2	"	"	12.6	515
7-F				139.8			580	530	4.43	5.8	22	30	18.8	575

‡ mean of
.4318
and
.3112

+ off tapes
Report TP 2627

+++ Supposed to be degrees using tape conversion eqn. supplied.

TABLE 4.9

1981 EARLY SEASON DATA

5 JULY 1981 - LANCASTER SOUND

ICE CONDITIONS	START TIME 17:28:13				FILE ME 8101.001		
	Elapsed Time Min.	Tp kN	Q kN.m	V Kn.	N RPM	P Deg.	Ri kN
18 Hours	0 15	120	130	6.0	112	12.4	10
	15 30	120	125	6.7	112	12.6	0
(1) 1856 Strips 1/10 Second Year Bergs & Growlers Slight Sea & Swell (2) 1/10 [2Y + MY] (3) 1/10 Pack (4) h < 0.6 m) σ < 100 kPa)	30 45	120	120	6.9	112	12.6	0
	45 60	110	120	6.6	112	12.6	0
	60 74	120	120	6.4	112	12.4	0
	75 90	240	225	9.5	125	17.0	30
	90 105	220	220	10.0	126	17.2	0
	105 120	200	210	9.8	133	15.4	0
	120 135	220	220	9.8	133	15.2	0
	135 150	290	280	11.4	142	17.2	0
	161 165	130	130	5.2	118	11.6	30
20 Hours	165 180		Variable				
	180 195		Variable				
	195 210		Variable				

(1) Chief Officers Log Book
(2) Master Data Package
(3) TP3632 Vol. 1
(4) TP3632 Vol. 2

Av. $V = 8.0$ Kn.
Av. $R_i = 6.4$ kN
Av. $R_{cw} = 137$ kN
Av. $\frac{R_i}{R_{cw}} = 4.7\%$

TABLE 4.10

1981 EARLY SEASON DATA

9 JULY 1981 - APPROACH TO NANISIVIK

ICE CONDITIONS	START TIME 0:18:44				FILE ME 8108.001		
	Elapsed Time Min.	Tp kN	Q kN.m	V Kn.	N RPM	P Deg.	Ri kN
(2) Level Ice h=122 m (3) Level Ice h=122 m	0 10	910*	735*	1.2	143*	22.8*	1,550*
(4) Level 1st Year Ice h = 1.38 m) σ = 100 kPa) 01 Hours	START TIME 0:30:18				FILE ME 8108		
	1 15	890	710	2.3	143	22.6	1,520
	15 26	900	715	1.9	143	22.6	1,560
	30 45	Manoeuvring Alongside					
	45 60				Shut Down		

* Oscns. at 2 Cycs/Min.

TABLE 4.11

1981 EARLY SEASON DATA

12 JULY 1981 - ADMIRALTY INLET

ICE CONDITIONS							
		START TIME 6:21:42				FILE ME 8109.001	
	Elapsed Time Min.	Tp kN	Q kN.m	V Kn.	N RPM	P Deg.	Ri kN
(1) 06 Hrs. Clear of Pack 08 Hrs. Bergs & Growlers	0 2	500	400	8.5	139	19.4	450
07 Hours		START TIME 7:14:29				FILE ME 8109.002	
(2) Broken Channel	0 1	320	290	11.6	134	18.8	15
(3) Broken Channel		START TIME 7:14:49				FILE ME 8109.003	
	0 3	320	290	11.6	134	18.8	15
		START TIME 7:19:12				FILE ME 8109.004	
	0 2	320	300	11.7	134	19.0	15
		START TIME 7:21:55				FILE ME 8109.005	
	0 11	320	295	12.0	134	19.0	0
		START TIME 7:33:44				FILE ME 8109.006	
	0 15	34	300	11.7	134	19.0	30
08 Hours	15 30	380	320	10.0	134	19.0	200
	30 37	550	440	10.5	144	20.4	390

Av. $V = 10.95$ Kn
Av. $R_i = 139$ kN
Av. $R_{cw} = 250$
Av. $\frac{R_i}{R_{ow}} = 55.8\%$

TABLE 4.12

1981 EARLY SEASON DATA

12 JULY 1981 - ADMIRALTY INLET

ICE CONDITIONS	START TIME 8:27:47					FILE ME 8110.001	
	Elapsed Time Min.	Tp kN	Q kN.m	V Kn.	N RPM	P Deg.	Ri kN
(1) 08 Hrs. Bergs & Growlers	0 15	580	450	9.7	144	20.4	480
(2) Closed Channel 09 Hours	15 30	860*	650*	4.4	144*	22.0*	1,260
(3) Some Movement of Ice Already. Track Closed Ship Broke New Passage	30 45	870*	660*	4.0	144*	22.4*	1,290
	45 60	850†	660†	6.8	145	22.4†	1,080

* Oscns. at 2 Cycs./Min.
* Do.
† Some Oscns.

TABLE 4.13

1981 EARLY SEASON DATA

12 JULY 1981 - OFF CAPE FAREWELL

ICE CONDITIONS	START TIME 16:18:56					FILE ME 8111.001	
	Elapsed Time Min.	Tp kN	Q kN.m	V Kn.	N RPM	P Deg.	Ri kN
(1) Reduce Spd. Ice Strip 6/10 [2 Yr. + MY]	0 15	220	-	3.7	122	14.4	260
(2) Pack Ice	15 24	220	-	4.0	121	14.4	240
17 Hours	START TIME 17:2:31					FILE ME 8111.002	
	0 4	100	-*	4.6	108	10.4	30

Av. V = 4.1 Kn.
Av. R_i = 177 kN
Av. R_{cw} = 50 kN
Av. $\frac{R_i}{R_{ow}}$ = 353%

TABLE 4.14

SUMMARY TABLE M.V. ARCTIC BROKEN ICE RESISTANCE

Date	Location	*1 Table #	Ship Velocity kt	*2 Ice Concentration MY/10	IY/10	Ice Thickness cm	Ice Resistance kN -/Avg.	Comments
5/7/81	Lancaster Sound	14	6 - 11		1	<60	0-30/6	
12/7/81	Admiralty Inlet	16	8 - 12	?	9+	?	15-450/159	
12/7/81	Admiralty Inlet	17A	4	-	9+	?	1260	
12/7/81	C. Farewell	17B	4	-	6+ 1	?	250 30	
3/11/81	Baffin Bay Baffin Bay	25	15 15	- -	9+ 7+	15 - 20 10	15-120/74 45-75/60	
3/11/81	Baffin Bay Baffin Bay	25	6 - 9	-	2 - 8	0 - 40	50-475/190	
4/11/81	Lancaster Sound	27	6 - 15	Trace	1 - 2	30	0-305/69	
4/11/81	Lancaster Sound	28	3 - 8	-	1	30	120-215/162	
4/11/81	Lancaster Sound	28	3 - 4	3	6	30 - 120	160-165/160	
4/11/81	Lancaster Sound	29	3	3	6	30 - 120	150-265/201	
6/11/81	Lancaster Sound	32	6 - 8	Loose	Pack?	?	520	
7/11/81	Admiralty Inlet	33	4 - 5	2	5	20 - 480	175-/205 235	
8/11/81	Admiralty Inlet	35	2-	2	6	20 - 480	900-/1033 1160	
8/11/81	Admiralty Inlet	36	5 - 10	4	4	20- 480	285-/400 500	
8/11/81	Admiralty Inlet	37	10	-	6	30	675	
8/11/81	Admiralty Inlet	38	11 - 14	-	9	30	50-/271 565	

SUMMARY TABLE M.V. ARCTIC BROKEN ICE RESISTANCE (Continued)

Date	Location	*1 Table #	Ship Velocity kt	*2 Ice Concentration MY/10	IY/10	Ice Thickness cm	Ice Resistance kN -/Avg.	Comments
11/11/81	Admiralty Inlet	39	4 - 12	Trace	9	?	220-/321 460	
12/11/81	Lancaster Sound	41	4 - 6	3	7	10 - 60 120 - 180	85-265/143	Following Louis
15/11/81	Pond Inlet	47	2 - 7	1	9	20 - 30 30 - 180	785-1170/ 977	Follow Louis
15/11/81	Pond Inlet	48 A/B	5 - 10	1	9	20 - 30 -180	100-465/285	Follow Louis
15/11/81	Baffin Bay	49	4 - 10	1	8	15 - 20	85-1035/381	Follow Louis

Notes:

1 Table # refers to tables of 1981 voyage data prepared by Coldstream Maritime Inc.

2 Larger ice thickness range refers to separate records of second or multi-year ice where available.

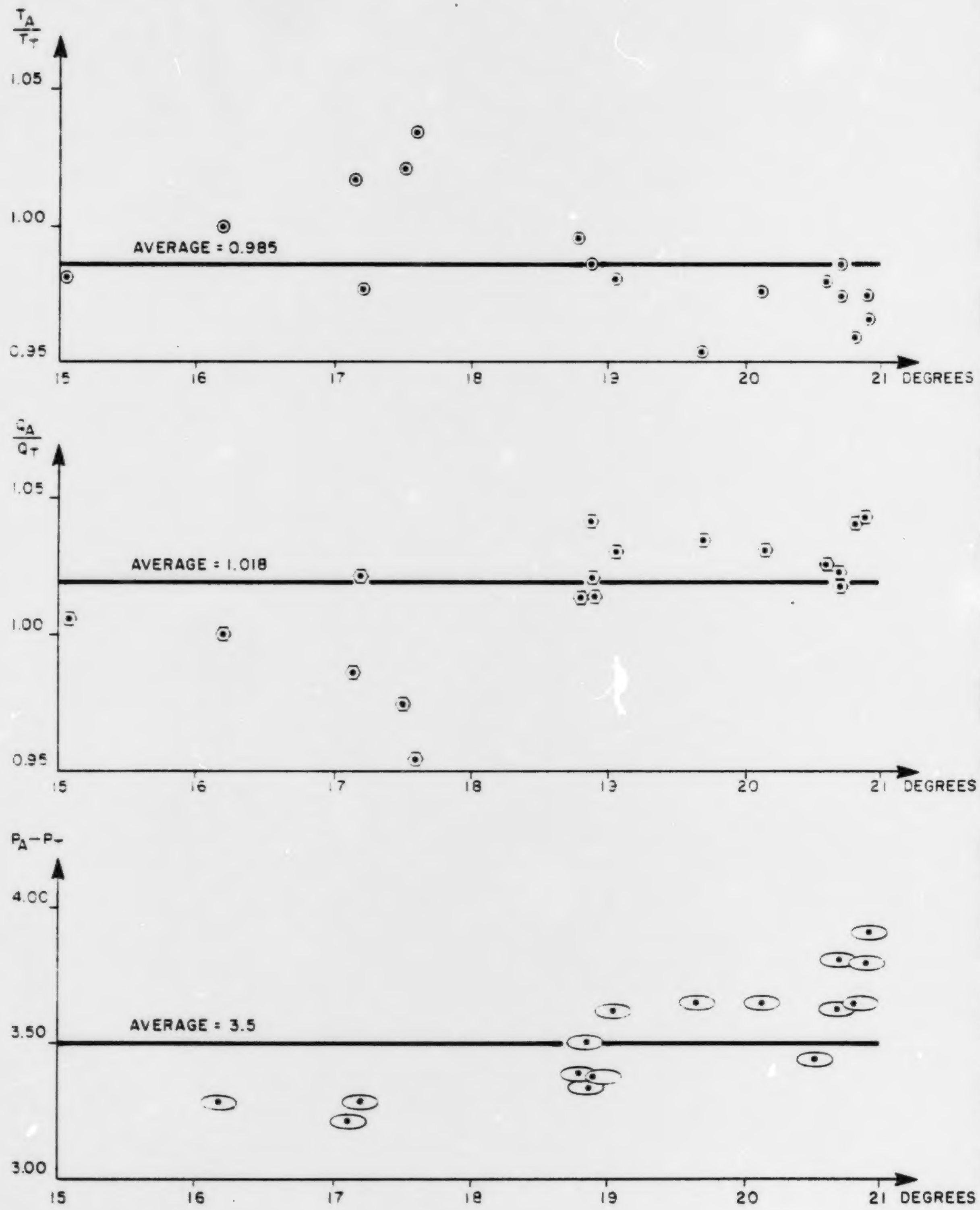


FIGURE 4.1 THRUST, TORQUE AND PITCH CORRELATION

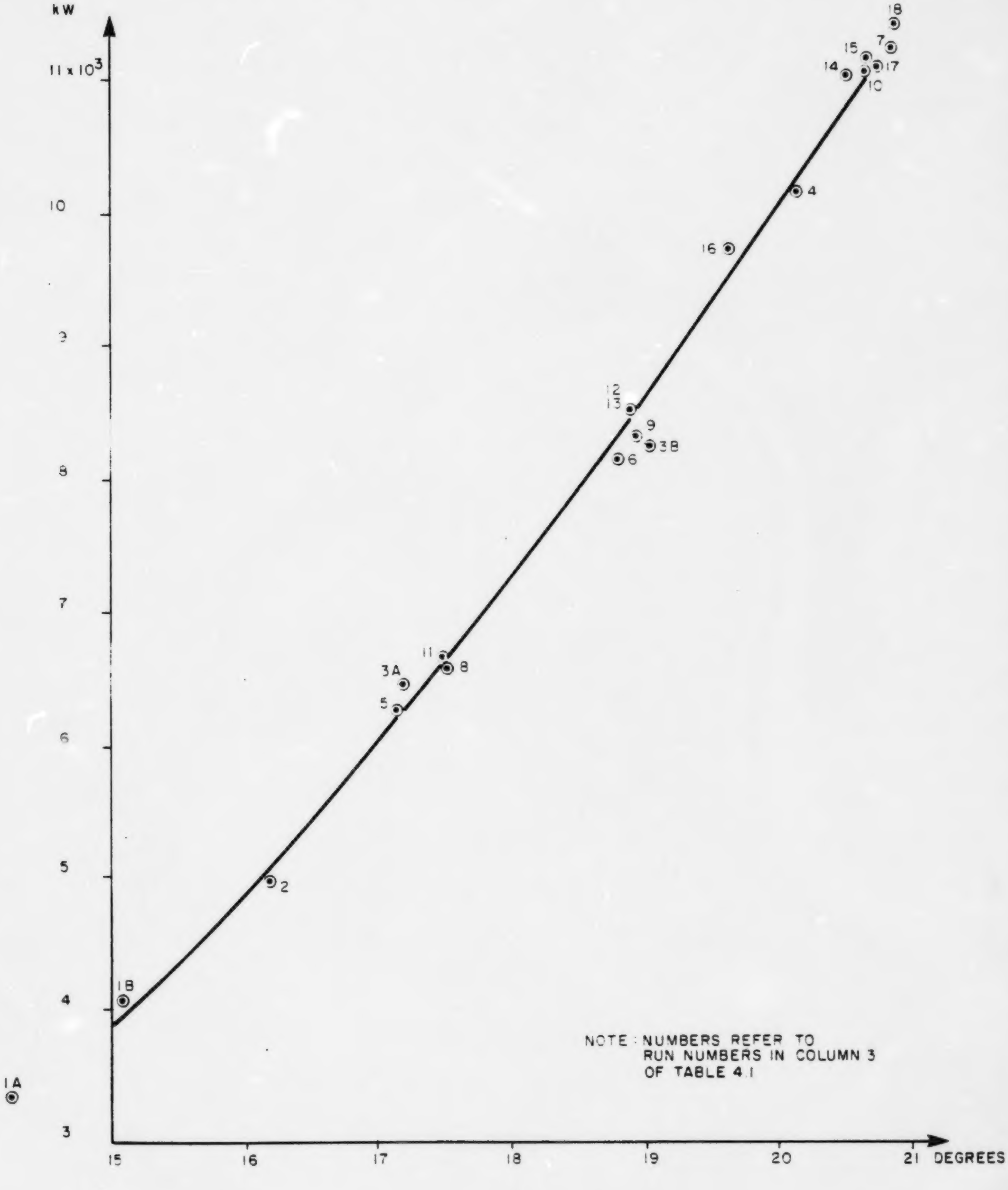


FIGURE 4.2 COMBINATOR EFFECTS. SHAFT POWER VS. PITCH

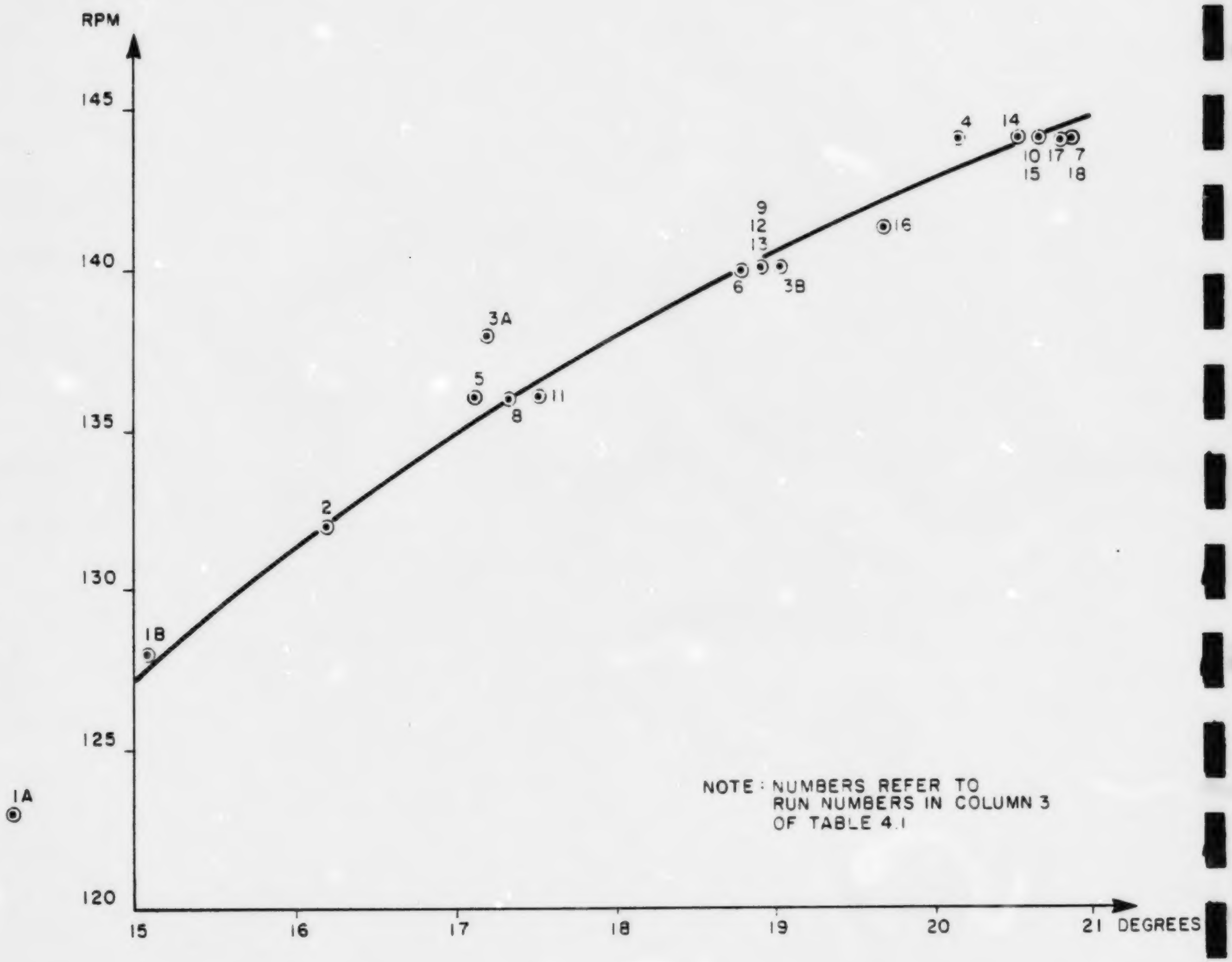


FIGURE 4.3 COMBINATOR EFFECTS. SHAFT SPEED VS. PITCH

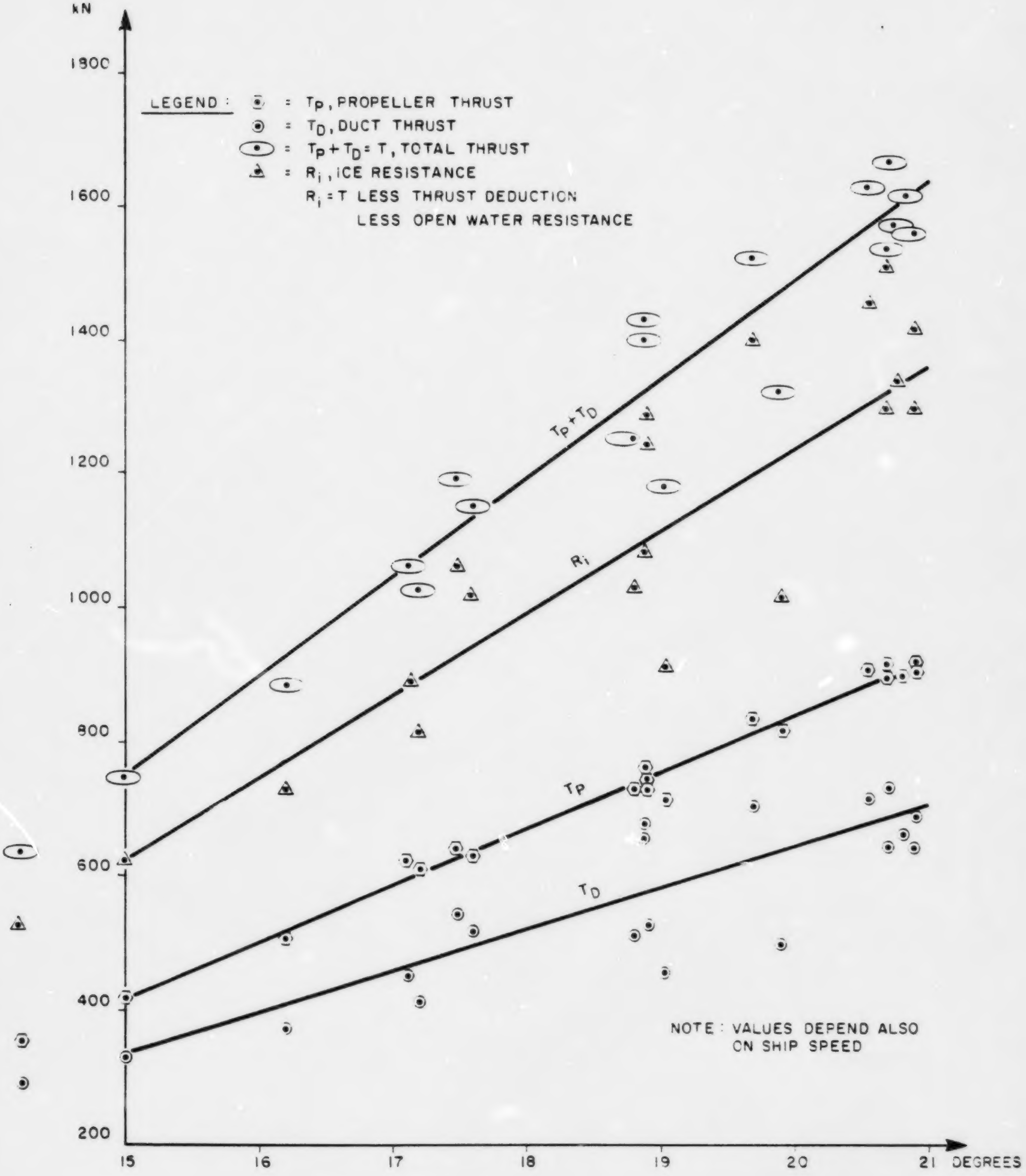


FIGURE 4.4 THRUST OR RESISTANCE VS. PITCH

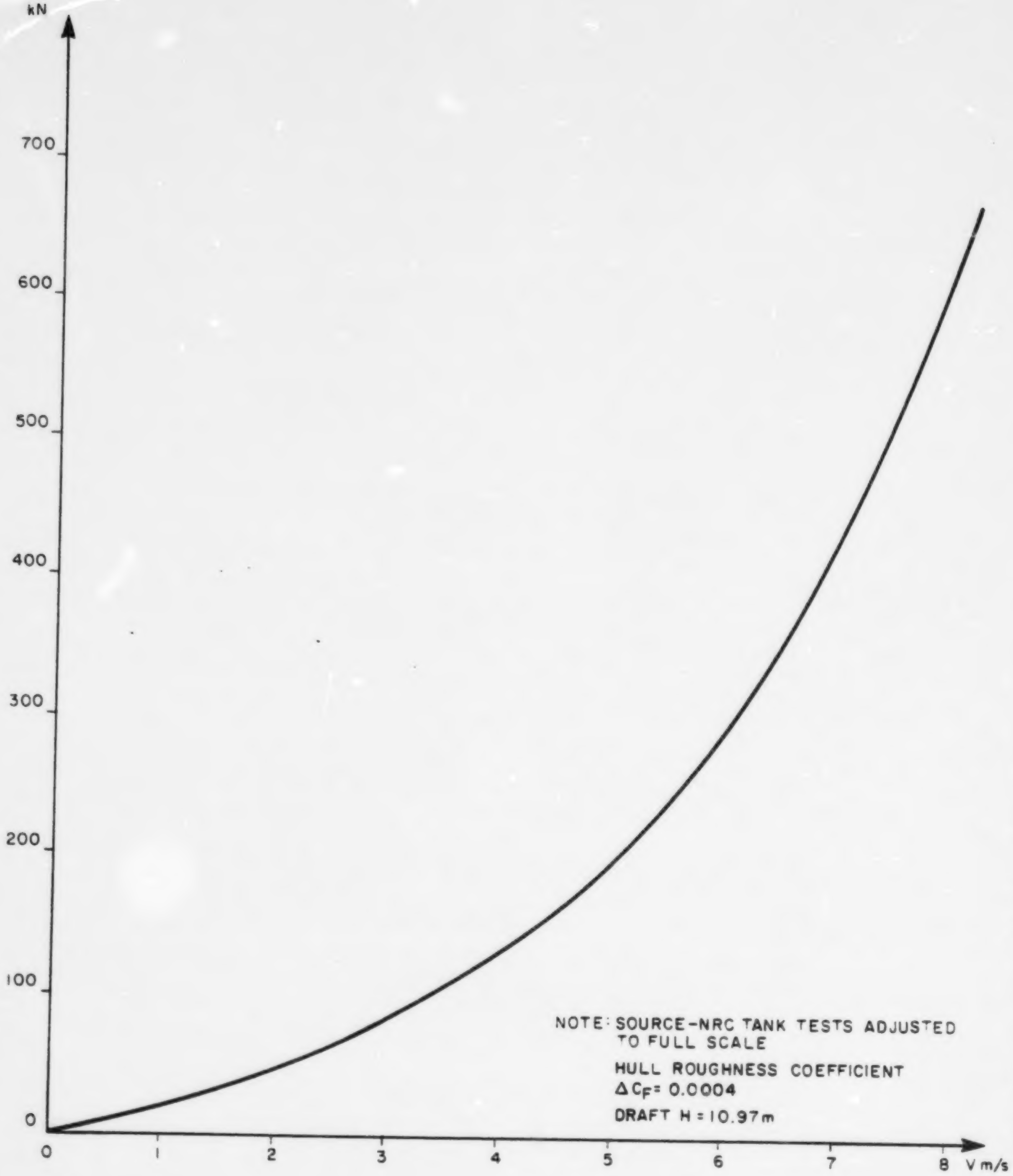


FIGURE 4.5 OPEN WATER RESISTANCE

5.0 DEVELOPMENT OF PERFORMANCE FORMULAE

5.1 General

As demonstrated in Section 2 of this report, the development of general equations to predict the performance of ships in various types of ice is still an inexact science and there is still no general agreement regarding the form such equations should take. The main advance in recent years is the realization that the icebreaking component of total resistance is much greater than previously thought and that friction plays an important part in relation to the shape of the bow, particularly in the area of the shoulders.

A special problem in attempting to formulate a general equation from the M.V. Arctic data is that the range of ice thickness and speed in level sheet ice tests is fairly small, thus giving an unreliable base for extrapolation. In the case of broken channels and pack ice, ice thickness, ice concentration and floe size is not generally given, thus making it difficult, if not impossible, to derive a meaningful equation representing performance. Peformance in broken channels and pack ice has, therefore, to be related to a general description of the ice in terms of location, time of year, probable thickness of ice and estimated concentration.

Since resistance to the motion of a ship in continuous sheet ice has received much more study and since the available data is more complete and precise, emphasis is placed on this ice condition in the following discussion.

As discussed in Section 2, it is possible to take any equation of the Kashtelyan form and, by regression analysis or trial and error, modify the coefficients and indices to obtain a good fit for the test data. However, such an equation is likely to be limited to the range of test data and cannot safely be used for extrapolation outside the test data or application to a vessel of different size or shape.

In the present state of the art and, in view of the large number of variables involved in ship ice interaction, it is probably not possible to derive a single all-embracing equation that will apply to all sheet ice conditions and to all sizes and shapes of vessel. It is also not within the scope of this study to attempt such a task. It is, nevertheless, desirable to put forward an equation which not only fits the trial data of the M.V. Arctic but also has a sound fundamental analytical basis that will give confidence for some extrapolation to greater ice thicknesses, higher velocities and other ships of different sizes.

The three well-known analytical equations of Kashtelyan, Enkvist and Poznyak are reviewed in this Section together with an entirely new form of equation recently developed by Carter. The equations are tested against the available M.V. Arctic trials data and a modified form of the Poznyak and Ionov equation (Eqn. 10 Table 5.2) is tentatively selected as an equation to predict the performance of the M.V. Arctic in continuous sheet ice.

5.2 Analysis of Data from Trials in Level Sheet Ice

The extraction of data from the dedicated trials and its presentation in useful form is discussed in Section 4 and summarized in Tables 4.1 to 4.9. The ice resistance, R_i, which excludes the open water resistance is plotted against velocity for various ice thicknesses in Figures 5.1, 5.2 and 5.3 for Lake Melville, Baffin Island and Saguenay River Trials respectively.

In order to compare equations of the Kashtelyan types, it is also helpful to plot the data in other ways.

The equations of Kashtelyan, Enkvist and Poznyak, although varying substantially in detail, are of the same basic form.

$$R_i = R_b + R_s + R_v \text{ ------------ } (1)$$

With the parameters usually assumed as affecting ice resistance, equation (1) can be written in general terms as:

$$R_i = C_b \sigma_f \; B^x h^a + C_s \gamma_i \; B^y h^b + C_v \gamma_i \; B^z h^c \; (F)^m \text{ ----- } (2)$$

Where F is the Froude number, usually $V/\sqrt{gh}$, and the indices a, b, c, x, y, z and m should be such as to give units of force for each term. However, different experts ascribe different values to the indices and, before reviewing specific equations, it is desirable to make some dimensionless and logarithmic plots of the M.V. Arctic data to establish indices of the variables h and V as closely as possible.

Figures 5.4, 5.5 and 5.6 show dimensionless plots of all trials data with bubbler off in the form of

$$\frac{R_i}{\gamma_i h^2 B} \quad \text{vs.} \quad \frac{V}{\sqrt{gh}}$$

$$\frac{R_i}{\gamma_i h^{1.5} B H^{0.5}} \quad \text{vs.} \quad \frac{V}{\sqrt{gh}}$$

$$\frac{R_i}{\gamma_i h^{1.5} B H^{0.5}} \quad \text{vs.} \quad \frac{V}{\sqrt{gL_w}}$$

Where L_w is the cube root of the volumetric displacement

The conclusions to be drawn from these plots are:

(1) That ice resistance varies approximately linearly with velocity, as assumed by most experts.

(2) Plots of $\frac{R_i}{\gamma_i h^{1.5} BH^{0.5}}$ show Lake Melville data averaging above or on the same line as Baffin Island data whereas plots of $\frac{R_i}{\gamma_i h^2 B}$ show Lake Melville data generally below. In view of the fact that Lake Melville ice strengths were substantially below those measured in the Baffin Island tests the h^2 relationship would appear to be more nearly correct for the non-velocity components of resistance.

(3) The thin ice test data from Navy Board Inlet are so far from the general trend that some special explanation is required for the excessive resistance which appears to be largely dependent on velocity. This should be a matter for further study.

(4) The general equation for R_i, assuming $F = \frac{V}{\sqrt{gh}}$, is $\frac{R_i}{Ch^n} = A + \frac{V}{\sqrt{gh}} \tan\theta$ where A is the vertical intercept of the trend line and θ its angle.

For a particular ice strength and friction:

$$R_i = CAh^n + \frac{CV}{gh} h^{n-0.5}$$

$$= C_1 h^n + C_2 V h^{n-0.5} \quad \text{-------------------------- (3)}$$

When V is very small, the velocity term is small and $R_i = C_1 h^n$ approximately $\therefore \log R_i = \log C_1 + n\log h$ ---------- (4)

A plot of R_i against h for small velocities on logarithmic paper will indicate the index n by the angle of the trend line. This is shown in Figure 5.7 which suggests an index of about 1.80 after adjusting the Lake Melville ice strength to the same value as that for Baffin Island and assuming that R_b is about 70% of $(R_b + R_s)$ and varies directly with strength. The Saguenay River data have been shown but largely ignored because the ice strength has not been directly measured. However, in spite of the large scatter in the low velocity Saguenay River data, Figure 5.7 indicates that the ice strength is approximately the same as in the Baffin Island trials.

For a constant ice thickness, h, equation (3) may be written

$$R_i = K_1 + K_2 V^m$$

$$\therefore \log (R_i - K_1) = \log K_2 + m\log V \text{ ----------- } (5)$$

$R_i - K_1 = R_v$ and a plot of R_v versus V on logarithmic paper will verify the value of the index m and show approximate values of K_2. Assuming values of K_i of 400 and 175 from Figures 5.1 and 5.2, log (R_v) is shown plotted against log V in Figure 5.8 for a Lake Melville ice thickness of 0.64 m and a Baffin Island ice thickness of 0.33. These plots indicate a variation of resistance with velocity which is slightly greater than linear. These plots also indicate the velocity components of resistance for ice thicknesses 0.64 m and 0.33 m at low speeds which enables a rough zero speed adjustment to be made as shown to Figure 5.7.

From the above analysis there is strong evidence that the sum of the non-velocity components of resistance, $R_b + R_s$, varies with the ice thickness, h, to a power between 1.5 and 2.0. Since most forms of equations have one of the two non-velocity dependent terms governed by h^2 and the other by h there is also evidence that the h^2 term should predominate.

With regard to the velocity dependent term R_v, the evidence is that this term varies approximately linearly with V. There is not enough data to establish the variation of R_v with h but inspection of the change of slope in the trend lines for h = 0.64 m, 0.33 m and 0.44 m in Figures 5.1, 5.2 and 5.3 indicates that R_v may vary with a higher power of h than the linear relationship usually assumed, unless other variables such as E or σ are introduced into the velocity dependent term in such a way as to give a similar effect. This is discussed later.

The various predictor equations already put forward for the performance of the M.V. Arctic and the other variations of established analytical equations are examined in the following sub-sections and tested against the data.

When considering the fit of these equations it should be borne in mind that the whole hull of the M.V. Arctic was painted to draft line 35 feet with the low friction solvent-free epoxy coating Inerta 160 in May 1980 and was touched up as necessary when dry-docked in June 1982. The resulting hull friction is not known but tests by others (see Table 2.2) indicate that dynamic friction would be in the range of 0.015 to 0.056 without snow cover and 0.08 to 0.15 with snow cover.

5.3 Existing Predictor Equations for M.V. Arctic

The predictor equations previously derived by others are shown in Table 5-1. They are compared in Figures 5.9 and 5.10 with the data from the Lake Melville and Baffin Island trials, plotted in both cases as total resistance (including open water resistance) against velocity.

It can be seen that Equation 1, based on 1974 model tests, overestimates the resistance at higher ice thicknesses and medium velocities, even with a dynamic coefficient of friction as low as 0.03, which would be a reasonable value with an epoxy coating in

TABLE 5-1

M.V. ARCTIC

RESISTANCE PREDICTOR EQUATIONS DERIVED BY OTHERS

Equn. 1. **MODEL TESTS**

from Arctec Canada Ltd. report - reference

$$R_t = 0.523\mu\sigma Bh + 4.90\rho_w gBh^2 + 10.42\rho_w\ Bv\ \sqrt{gh^3} \quad \text{-------------} \ (1)$$

modified Kashtelyan equation in all three terms supported by multiple regression.

Equn. 2. **SHIP TRIALS**

from German & Milne report - reference TP 2627 Vol. II

$$R_t = 0.004\mu_o\sigma Bh + 3.6\mu_o\rho_i Bh^2 + 0.0066\,\eta_2^{-1} \cdot B^{1.65}hV + R_{ow} \quad \text{------} \ (2)$$

modified Kashtelyan equation in third term only.

Equn.3. **SHIP TRIALS**

from Melville Shipping Ltd. report - reference TP 3646 Vol. I

$$\begin{aligned} R_t = {} & [2/3 + \sigma/900] \cdot [0.5937h^{2.051} + 0.5315h^{1.082}] + \\ & + 0.1898Vh^{1.436} + 0.0001626V^3 \quad \text{----------} \ (3) \end{aligned}$$

based on Poznyak and Ionov.

	UNITS ADOPTED		
LEGEND	Equn. 1	Equ. 2	Equ. 3
R_i = icebreaking	Newtons	Tons	MN
R_t = total resistance	"	"	"
σ = ice flexural strength	Pascals	Tons/ft.^{2}	kN/m^2
h = ice thickness	metres	feet	metres
ρ = mass density, water or ice	kg/m^3	Tons/ft.^{3}	-
g = gravitational acceleration	m/s^2	ft./sec.^{2}	-
B = ship beam	m	feet	metres
V = ship speed	m/s	ft/sec	knots
μ, η ship friction and shape parameters: -			
μ_o	-	1.65	-
η_2	-	2.62	-

good condition. Equations 2 and 3 both overestimate the resistance, particularly for the Baffin Island trials, but Equation 3 has a good shape and would fit quite well if the non-velocity component is reduced. Overall, Equation 3 is the best fit.

It must be realized that Equation 1 is based on the Kashtelyan equation but modified by regression analysis of limited data and cannot safely be extrapolated beyond the range of ice thickness and speed tested or to a ship of a different size or shape. This is illustrated by Figures 5.11 and 5.12 which show how far Equation 1 diverges from Equations 2 and 3 over a greater range of speed and ice thickness. All three equations will also give substantially different predicted resistance for vessels of different beams at high velocities since they have B to different powers, in the velocity dependent term.

5.4 Review of Selected Analytical Equations

Based on research to date, it would appear that any general resistance equation applicable to a range of vessel shapes and sizes as well as a range of ice thicknesses and speeds should probably embody the following parameters and coefficients:

ice thickness,
speed of ship,
beam of ship,
length of ship or forebody,
draft of ship,
bow opening angle
stem angle
flare angles
coefficient of friction, between ship or ice
  including effect of snow cover
ice flexural strength
ice strain modulus
density of ice) and/or difference between densities
density of water)

For a proper comparison of model and full-scale tests, the powers or indices which are applied to the parameters, h, V, B, L and H should be such that the equation can be written in the form of dimensionless ratios. The displacement of the ship will also be a factor when considering the powering requirements for transiting ridges or other ice conditions giving local increases in resistance.

Apart from the long standing equations of Kashtelyan[1] and Enkvist[12], interesting equations based on fundamental analysis and full scale tests have recently been put forward by Poznyak and Ionov[24] and by Carter[41]. These equations, amongst others, are tested in the following pages for this ability to predict the performance of the M.V. Arctic.

5.4.1 Kashtelyan

The equation of Kashtelyan is summarized in Appendix "A" and full details can be found in Reference 1. Kashtelyan's equation has been used as the basis for most equations produced since by others. In its final simplified form the equation becomes

$$R_t = 0.004\ B\sigma_f h\mu_0 + 3.6\gamma_i\ Bh^2\mu_0 + 0.25\ B^{1.65}\ \frac{hV}{\eta_2} + R_{o\omega} \text{ ------------- } (4)$$

The above equation is in metric units and R_t, γ_i and σ are in tonnes force. The third term is not dimensionally correct and has the dimensions of $L^{3.65}/T$. To transfer from metric to feet units the coefficient of 0.25 must be divided by $(3.28)^{3.65}$ giving a coefficient of 0.0033. In equation 2 of Table 5.1 this coefficient has been arbitrarily doubled to 0.0066 in order to make a good fit of the M.V. Arctic data in conjunction with values of 1.65 and 2.62 for μ_0 and η_2 respectively.

Kashtelyan derived his numerical coefficients mainly from model and full-scale tests of the Russian Icebreaker 'Ermak'. A more analytical approach to modifying the coefficient for the third term is to examine the differences in the dimensional ratios of the 'Ermak' and the M.V. Arctic which are as follows:

	Ermak	M.V. Arctic
Length, L, at waterline	92.5	196.6
Beam, B, at waterline	21.5	22.9
Draft, H	7.6	10.97
L/B ratio	4.3	8.6
B/H ratio	2.83	2.09

There is strong evidence that resistance increases with draft and logical argument that this should be associated with velocity since the increased velocity of the hydraulic flow lines round the hull will drag more ice to lower depths. A reasonable modified third term incorporating draft which is also dimensionally correct would be:

$$C_v B H^{0.5} \gamma_i h^{1.5} \frac{V}{\sqrt{gh}} \cdot \frac{1}{\eta_2}$$

The effect of using $BH^{0.5}$ instead of $B^{1.65}$ will be as follows:

	Ermak	M.V. Arctic
$B^{1.65}$ =	158	175
$BH^{0.5}$ =	59.3	75.8
Ratio $B^{1.65}/BH^{0.5}$ =	2.66	2.31

Therefore, in order to suit the Ermak, the third term coefficient in equation (4) will have to be increased 2.66 times from 0.25 to 0.66, which will have the effect of giving a predicted increase for the M.V. Arctic of 2.66/2.31 compared with a term using $B^{1.65}$. The corresponding value of C_v is $0.66 \times \sqrt{g}/\gamma_i$ and the general equation becomes:

$$R_t = 0.004\ B\sigma_f h\mu_o + 3.6\gamma_i\ Bh^2\mu_o + 2.2\ BH^{0.5}\gamma_i h^{1.5} \cdot \frac{V}{\sqrt{gh}} \cdot \frac{1}{\eta_2} + R_{ow} \quad \text{-- (5)}$$

The coefficients μ_o and η_2 are shape factors only and friction is not specifically taken into account in the equation which assumes a typical hull surface roughness without a special coating such as Inerta 160. An adjustment should, therefore be made for the low friction hull coating applied to the M.V. Arctic.

A means of adjusting friction has recently been derived by both Melville Shipping Ltd. and by Arctec Ltd. who have produced new geometry-friction coefficients μ_1 and μ_2 which are related to the Shimansky coefficients μ_o and n_2 so that at zero friction

$$\mu_1 = \mu_o - 1 \quad \text{and} \quad \mu_2 = 1/n_2$$

The values of μ_1, μ_2 derived by Arctec and the corresponding values of μ_o' and n_2' taking friction into account are as follows:

f_d	μ_1	μ_2	μ_o'	n_2'
0.0	0.67	0.40	1.67	2.48
0.1	0.86	0.55	1.86	1.83
0.2	1.05	0.71	2.05	1.41
0.3	1.25	0.88	2.25	1.13

The above values of μ_o' and n_2' cannot, however, be applied directly to equation (5) because they are based on a zero friction relationship with the original Shimansky coefficients. Assuming that the typical icebreakers on which Kashtelyan based his numerical coefficients had a dynamic hull/ice friction of 0.1 (see Table 2.3), the numerical coefficients governing the first two terms should be reduced in the ratio 1.67/1.86 and that of the third by 1.83/2.48. Equation 5 becomes therefore:

$$R_t = 0.0036\ B\sigma_f h \mu_o' + 3.23 \gamma_i B h^2 \mu_o' + 1.62\ BH^{0.5} \gamma_i h^{1.5} \cdot \frac{V}{\sqrt{gh}} \cdot \frac{1}{n_2'} + R_{ow} \quad \text{--(5a}$$

If this equation is applied to the M.V. Arctic using 0.94 t/m^3 for γ_i

$$R_t = 0.082\,\sigma_f h \mu_o' + 69.7h^2 \mu_o' + 36.8hV/\eta_2' + R_{ow} \tag{6}$$

Figure 5.13 shows R_t plotted in kiloNewtons against V for ice thicknesses of 0.61 and 1.0 metres for values of f_d of 0.03, 0.08 and 0.13. Figures 5.17 and 5.18 compare the modified Kashtelyan equation with other selected equations for accuracy of fit with the Baffin Island and Saguenay River trials data, assuming $f_d = 0.03$. The modified Kashtelyan equation fits well with the data from both Lake Melville and Baffin Island.

5.4.2 Enkvist

The Enkvist equation is summarized in Appendix "A" and described in detail in Reference 12. When the dimensions of the M.V. Arctic and its bow angles are inserted and the correlation factors of 2.0 are applied to the breaking and submersion terms, the equation reduces to the following in Newtons with σ and E in Pascals.

$$\begin{aligned} R_t = {} & 265 \frac{\sigma_f^2}{E} h \left\{ 1 + \frac{4.41 f_d}{1 - 1.1 f_d} \right\} + 394{,}000h\,(0.7 + 0.854 f_d) + \\ & + 575{,}000\,h \left(0.744\,h + \frac{V^2}{39.2} \right) \left\{ \frac{0.227 + 0.75 f_d}{0.393 - 0.43 f_d} + 0.255 + 0.95 f_d \right\} \\ & + 15{,}050\,hV^2 \left\{ 1 + \frac{4.41 f_d}{1 - 1.1 f_d} \right\} + R_{ow} \end{aligned} \tag{7}$$

For a dynamic friction coefficient, f_d of 0.03, the equation reduces to the following in kiloNewtons with σ in kPa.

$$R_t = 301 \frac{\sigma_f^2}{E} h + 286\,h + 402\,h^2 + 30.9\,V^2 h + R_{ow} \tag{8}$$

The results of the equation for ice conditions encountered in the Lake Melville trials are plotted in Figure 5.14 for ice thicknesses, h, of 0.61 and 1.0 metres and for three different friction coefficients, f_d, of 0.03 and 0.08 and 0.13.

The generally low resistance given by the Enkvist equation is probably mainly due to the very small value given by the first term of the equation governed by σ_f^2/E and Enkvist states in his treatise[12] of 1972 (page 147) "There is no doubt that the breaking resistance is a very small component in full scale." This does not agree with recent research and model tests by Poznyak and Ionov[24] indicate that the breaking resistance for the medium size icebreaker tested is about 47% of the total direct non-velocity dependant resistance with friction providing a further 34%. It appears therefore, that for more reliable prediction, the Enkvist equation requires substantial modification in the first term, and Enkvist, himself, now admits this.[28]

5.4.3 Poznyak and Ionov

Following model tests to divide icebreaker resistance into components, Poznyak and Ionov developed a new analytical equation which is summarized in Appendix "A" and Reference 24. The equation is based on the earlier work of Kashtelyan, Poznyak and Jaryvlin but includes the friction coefficient and new bow angle relationships in each of the three resistance terms. The simplified equation may be written:

$$R_t = 0.014\ \sigma_f\ h^2 \{\mu_1\} + 0.1\ \gamma_\Delta\ h\ BL\{\mu_2\} + 1.5\ \gamma_i\ hB^2F_L\{\mu_3\} + R_{ow} \quad \text{---}\ (9)$$

where μ_1, μ_2, and μ_3 are coefficients determined from the bow angles, ship dimensions and the hull friction coefficient. F_L is the length Froude number $V/\sqrt{gL}$. It should be noted that μ_1 is dimensional and consists of $\mu_{1a}B + \mu_{1b}L$, as can be seen from Appendix A 2.25.

As in the case of the Kashtelyan equation, the Poznyak and Ionov equation is based on tests of typical icebreakers and tends to give a velocity component of resistance which is too low for cargo vessels. It can be seen that the third (velocity) term includes B^2 where Kashtelyan uses $B^{1.65}$ and most other authorities use B. In addition, the Froude number is $V/\sqrt{gL}$ which will give a reduced resistance for a vessel with a large L/B ratio such as the M.V. Arctic. Finally, no account is taken of the effect of draft. It is therefore, believed that the equation can be made to have more general applicability if the third term is modified to:

$C_v \gamma_i hBL^{0.5} H^{0.5} V/\sqrt{gL} \begin{Bmatrix} \\ \mu_3 \end{Bmatrix}$ Where C_v is derived below -------- (9a)

A persuasive justification for the above modification is given by the fact that $BL^{0.5}H^{0.5}$ is not very different numerically from B^2 in the case of a typical icebreaker such as the Ermak but is very different in the case of an icebreaking bulk cargo vessel such as the M.V. Arctic. This is shown below:

	Ermak	CCGS St. Laurent	M.V. Arctic
Length at waterline, L	92.5	102	196.6
Beam at waterline, B	21.5	24.4	22.9
Draft, H	7.6	8.99	10.97
B^2	462	595	524
$B.L^{0.5}.H^{0.5}$	570	739	1062
Ratio: $B.L^{0.5}.H^{0.5}/B^2$	1.23	1.24	2.03

In other words, if the numerical coefficient of 1.5 in the third term of equation (9) is divided by 1.23, the modified coefficient of 1.22 will give the same result for a typical icebreaker but a velocity resistance for the M.V. Arctic which is 2.03/1.24 or 1.64 times greater. This increase in the third term fits well with the Lake Melville trials data, but is much too great to fit the Baffin Island trials data for which a modified numerical coefficient of 0.80 is required. This suggests that there may be something more fundamentally wrong with the velocity dependent term and this may be in the power of h. As indicated in sub-section 5.2, in the absence of any variable other than h and v in the velocity dependent term, the change in slope of the trend lines in Figures 5.1, 5.2 and 5.3 requires h to be raised to a power of at least 1.5.

Equation (9a) may be written without changing the power of h, -

$$R_v = C_v \gamma_i \; h^{3/2} B H^{0.5} \; V/\sqrt{gh} \; [\mu_3] \text{ ----------- } (9b)$$

If the power of h in R_v is changed to $h^{3/2}$ the equation can be written

$$R_v = C_v \gamma_i \; h^2 \; B \left(\frac{H}{L_w}\right)^{\frac{1}{2}} \cdot \frac{V}{\sqrt{gh}} \; [\mu_3] \text{ --------- } (9c)$$

where L_w is the cube root of the volumetric displacement and introduced to retain dimensional correctness. In the case of the M.V. Arctic $L_w = 33.36$ and $(H/L_w)^{\frac{1}{2}}$ is 0.573.

From the dimensionless plot of Figure 5.4, $\tan\theta = 3.55$ and

$$R_v = 3.55 \; \gamma_i h^2 \; BV/\sqrt{gh} \text{ ------------------------------- } (9d)$$

Equating (9c) and (9d)

$$C_v \cdot \left(\frac{H}{L_w}\right)^{\frac{1}{2}} \mu_3 = 3.55$$

$$\therefore \quad C_v = 3.55/0.573 \, \mu_3 = 6.20/\mu_3$$

If it is assumed that the dynamic hull/ice friction of the M.V. Arctic is 0.03, $\mu_3 = 0.84$ and $C_v = 7.38$, and the general equations for the velocity dependent resistance becomes:

$$R_v = 7.38\ \gamma_i\ h^2\ B\left(\frac{H}{L_w}\right)^{\frac{1}{2}} \cdot \frac{V}{\sqrt{gh}}\ [\mu_3] \quad \text{------------- (9e)}$$

or

$$R_v = 7.38\ \gamma_i\ h^{3/2}\ BH^{\frac{1}{2}}\ \frac{V}{\sqrt{gL_w}}\ [\mu_3] \quad \text{------------- (9f)}$$

The breaking and submergence resistance is given in general terms by:

$$R_b + R_s = C_b\ \sigma_f\ h^2\ [\mu_1] + C_s\ \gamma_\Delta\ h\ BL\ [\mu_2] \quad \text{--------------- (9g)}$$

From the vertical intercept of the non-dimensional plot of $R_i/\gamma_i\ h^2\ B$ against $V/\sqrt{gh}$ in Figure 5.4, $R_b + R_s = 7.65\gamma_i\ h^2\ B$ $= 1622\ h^2$

In the model tests, Poznyak and Ionov[24] found that, with a model ice thickness of 29 mm, R_b and R_s were respectively 47% and 18% of the direct non-velocity resistance with friction adding 35%. Allocating friction in proportion

$R_b = 47 + 25 = 72\%$

and $R_s = 18 + 10 = 28\%$

which is consistent with the finding of Enkvist.[12]

$\therefore\quad C_b\,\sigma_f\ h^2\,\mu_1 = 1168\ h^2$

and $C_s\,\gamma_\Delta\,hBL\,\mu_2 = 454\ h^2$

From which $C_b\sigma_f\mu_1 = 1168$ so that for a Baffin Island trials average, ice strength of 370 kPa and μ_1 of 95.2 ($f_d = 0.03$), $C_b = 0.033$.

Similarly, $C_s \gamma_\Delta BL\mu_2 = 454\ h$ so that for a typical Baffin Island trials ice thickness of 0.33 m, μ_2 of 1.656 and γ_Δ of 0.785, $C_s = 0.025$.

Similar calculations for Lake Melville trials with ice thickness 0.64 m and strength 267 kPa give $C_b = 0.032$ and $C_s = 0.033$. Obviously, with breaking resistance varying with h^2 and submergence resistance varying with h, there cannot be a constant ratio of R_B/R_s which must vary with h. The final values for the coefficients C_b and C_s have therefore been chosen by trial and error as 0.035 and 0.029 respectively. The full modified equation, therefore, becomes:

$$R_t = 0.035\ \sigma_f h^2 \{\mu_1\} + 0.029\gamma_\Delta\ hBL\ \{\mu_2\} + 7.4\gamma_i\ B\left(\frac{H}{L_w}\right)^{\frac{1}{2}} h^2 \cdot \frac{V}{\sqrt{gh}} \{\mu_3\} + R_{ow} \qquad \text{------ (10)}$$

When the hull dimensions for the M.V. Arctic and values of 0.785 kN and 9.26 kN for γ_Δ and γ_i are inserted and bow angles substituted in the expressions for $\mu 1$, $\mu 2$ and $\mu 3$, the equation becomes:

$$\begin{aligned} R_t = {} & 0.035\ \sigma_f\ h^2\ (74.3 + 698\ f_d) \\ & + 102\ h\ (1.49 + 5.54\ f_d) \\ & + 287\ Vh^{3/2}\ (0.775 + 2.19\ f_d) + R_{ow} \end{aligned}$$

For a friction coefficient, f_d of 0.03, the equation reduces to the following in kiloNewtons:

$$R_t = 3.33\ \sigma_f\ h^2 + 169\ h + 241\ Vh^{3/2} + R_{ow}$$

Figure 5.15 compares R_t with Lake Melville data for ice thicknesses, h, of 0.61 m and 1.0 m and for dynamic friction coefficients, f_d, of 0.03, 0.08 and 0.13. Figures 5.17 and 5.18 compare the modified P & I equation with other selected equations for fit with the Baffin Island and Saguenay River trials data, assuming $f_d = 0.03$.

It can be seen that the fit of the modified P & I equation is quite good with all trials data, except thin ice in Navy

Board Inlet if $f_d = 0.03$ which is a reasonable hull friction for a ship coated with Inerta 160 and in conditions of negligible snow cover.

5.4.4 Carter

In a report dated March 1983 to the Transportation Development Centre (TP-3679E) entitled "Ship Resistance to Continuous Motion in Level Ice", Donald Carter has made an entirely new analytical approach to the derivation of a predictor equation. It is beyond the scope of this study to comment in detail on the Carter equation but mention is made of it because the approach is interesting and the derived equation appears to be capable of giving fair results not only for the M.V. Arctic but also for a number of other icebreakers for which full-scale test data is available. The general equation is summarized in Appendix "B" and reduces to the following when the dimensions and bow angles of the M.V. Arctic are inserted and there is no internal pressure.

$$R_t = \left\{0.547 \frac{(0.271 + 0.866 f_d)}{(0.469 - 0.5 f_d)} \sigma h^2 + \frac{110 \sigma_f h^{1.25}}{E^{0.25}} (0.577 + 3.24 f_d)\right\}$$

$$X \left\{1 + \frac{0.038 V^2 E^{0.5}}{\sigma_f h^{0.5}}\right\}^{0.5} + 900 h_s + R_{ow} \text{ in kN} \text{ ----------------- } (11)$$

Where h_s is depth of snow cover in metres.

For $f_d = 0.03$ and no snow cover, the equation reduces to the following in kiloNewtons:

$$R_t = \left\{0.358 \sigma_f h^2 + \frac{74.2 \sigma_f h^{1.25}}{E^{0.25}}\right\} \left\{1 + \frac{0.038 V^2 E^{0.5}}{\sigma_f h^{0.5}}\right\}^{0.5} + R_{ow} \text{ --- } (12)$$

Figure 5.16 shows the results of the Carter equation applied to the Lake Melville tests for ice thicknesses, h, of 0.61

and 1.0 metres and for hull friction coefficients of 0.03, 0.08 and 0.13. Figures 5.17 and 5.18 compare the Carter equation with other selected equations for fit with the Baffin Island and Saguenay River trials data. It will be seen that, for the plotted friction coefficients, the Carter equation gives results that are too low for the Lake Melville data but that, for an f_d of 0.2 as assumed by Carter himself, the fit would be good. For the other data the non-velocity dependent resistance fits well but R_v is high in the case of Baffin Island and low in the case of Saguenay River.

One variable that can make a significant difference to the value of resistance obtained from the Carter equation is the strain Modulus E. For instance, the Carter equation will fit well with the Lake Melville test data with the following simultaneous values of f_d and E, assuming σ_f = 267 kPa.

Coefficient f_d	0.03	0.08	0.13	0.20
Modulus E, MPa	151	379	821	2071

Carter assumes σ_f = 380 kPa and E = 1900 MPa compared with the average values of σ_f = 267 kPa and E = 2320 MPa measured during the Lake Melville trials. During the October 1980 Baffin Island trials on the M.V. Arctic, σ_f varied from about 235 to 410 kPa and E varied from 400 to 1400 MPa. Obviously the big differences in the ratio E/σ_f from 1500 to 8700 will have a significant effect on the resistance given by the Carter equation. This is discussed further in Sub-section 5.5.

5.4.5 Mathews' Equation

Mathews has developed a general analytical equation based on dynamic rather than quasi-static considerations. This is described in more detail in Appendix "C" and the basic expression is given below in non-dimensional form:

$$\frac{R_i}{\sigma_f^{\frac{1}{2}} (\rho g h)^{\frac{1}{2}} Bh} = \frac{A}{\left\{ \left[\frac{E\rho}{\sigma_f^2} \right]^{\frac{1}{2}} V \right\}^{\frac{1}{2}}} + G \left\{ \left[\frac{E\rho}{\sigma_f^2} \right]^{\frac{1}{2}} V \right\}^{\frac{1}{2}} \text{ -------- } (13)$$

from which:

$$R_i = A \frac{(\rho g)^{\frac{1}{2}} \sigma_f Bh^{3/2}}{(E\rho)^{\frac{1}{4}} V^{\frac{1}{2}}} + G (E\rho)^{\frac{1}{4}} (\rho g)^{\frac{1}{2}} Bh^{3/2} V^{\frac{1}{2}} \text{ --------------- } (14)$$

where A and G are constants for a given hull form and friction and $\rho = \rho_i$.

Equation 13 can be written:

$$C_i = \frac{A}{V_i^{\frac{1}{2}}} + GV_i^{\frac{1}{2}}$$

where C_i is resistance coefficient and V_i is velocity coefficient.

$\therefore \quad C_i V_i^{\frac{1}{2}} = A + GV_i$
or $\quad K_i = A + GV_i$
or $\quad K_i = A + G_0 (1 + mf_d) V_i$ allowing for hull friction

From the model and full scale experimental data, Mathews obtains the following numerical values (Note that A and G are one tenth of the values given in Appendix C to allow for the use of the full beam B in equations 13 and 14 instead of b, defined as B/10):

$$A = 0.70 \qquad G_0 = 0.277 \qquad m = 13.8$$

therefore $K_i = 0.70 + 0.277 (1 + 13.8 f_d) V_i$

The equation fits with an average value of $f_d = 0.178$ for the 1974 model tests and for indicated values of $f_d = 0.022$, 0.065, 0.019 and 0.024 respectively for the Lake Melville, Lancaster Sound, Strathcona Sound and Saguenay River Trials with Bubbler off.

Because Mathews has derived the above specific values for the friction coefficient to suit the available data, his equation is not plotted for comparison with the other equations discussed previously. With the derived coefficients, Mathews' equation fits quite well over the limited range of the trials data. However, the equation will need to be tested over a greater range of data and for other ships of different sizes and shapes. This is beyond the scope of this study.

5.5 Selection of Equation for Performance in Level Ice

The equations discussed in the foregoing sub-section are of two basic types. Firstly, equations such as those of Kashtelyan, Enkvist and Poznyak which assume that the ice resistance consists of the three basic components due to breaking ice, turning and submerging ice and forces dependent on velocity. And, secondly, equations,such as those of Carter and Mathews, which do not divide the resistance into components and are derived from a combination of quasi-static and dynamic principles.

The simplified forms of the first type of equation tested are compared in Table 5.2 from which the fundamental differences can be clearly seen. Kashtelyan and Enkvist assume R_b varies with h while Poznyak assumes h^2. Kashtelyan assumes R_s varies with h^2, Poznyak assumes h, and, in effect, Enkvist has two terms one varying with h and the other varying with h^2 which in his analysis is included as part of the velocity dependent resistance. Kashtelyan and Poznyak assume that R_v varies linearly with V while Enkvist assumes V^2. Over the relatively small range of velocities and ice thicknesses that most full-scale tests have been carried out there is not usually enough data to prove

TABLE 5.2

Selected Ice Resistance Predictor Equation Forms

Author	Breaking R_b	Submerging R_s	Velocity R_v
1. Kashtelyan	$C_b\sigma_f Bh$	$+ \; C_s\rho_i gBh^2$	$+ \; C_v B^{1.65}hV$
2. Kashtelyan (Mod.)	$C_b\sigma_f Bh$	$+ \; C_s\rho_i gBh^2$	$+ \; C_v^1\rho_i gBH^{0.5}h^{1.5}\dfrac{V}{\sqrt{gh}}$
3. Enkvist[(1)]	$C_b\dfrac{\sigma_f^2}{E}Bh$	$+ \; C_s\rho_\Delta gBhH + C_s^1\rho_w gBh^2$	$+ \; C_v\rho_i gBh^2V^2/gh$
4. Poznyak et. al.[(2)]	$C_b\sigma_f h^2 (C_1B + C_2L) +$	$C_s\rho_\Delta gBhL$	$+ \; C_v\rho_i gB^2h\dfrac{V}{\sqrt{gL}}$
5. Poznyak (Mod. Eqn. 9a)	$C_b\sigma_f h^2 (C_1B + C_2L) +$	$C_s\rho_\Delta gBhL$	$+ \; C_v\rho_i gBL^{0.5}H^{0.5}h\dfrac{V}{\sqrt{gL}}$
6. (Alternative form)	$C_b\sigma_f h^2 (C_1B + C_2L) +$	$C_s\rho_\Delta gBhL$	$+ \; C_v\rho_i gBH^{0.5}h^{1.5}\dfrac{V}{\sqrt{gh}}$
7. Poznyak (Mod. Eqn. 10)	$C_b\sigma_f h^2 (C_1B + C_2L) +$	$C_s\rho_\Delta gBhL$	$+ \; C_v\rho_i gB\left[\dfrac{H}{L_w}\right]^{0.5} h^2 \cdot \dfrac{V}{\sqrt{gh}}$

NOTE: (1) The third term, $C_s^1\rho_w gBh^2$, is part of an expression derived by Enkvist for R_v which is not actually dependent on velocity.

(2) The first term is not dimensionally correct and, for comparison of full-scale with model tests should be replaced by a dimensionless coefficient such as $\dfrac{C_1B + C_2L}{L_w}$ where L_w is the cube root of the volumetric displacement.

which equation is the more correct and any of them can be made to fit reasonably well with a suitable adjustment of the friction factors and numerical coefficients that make up the final values of C_b, C_s and C_v.

The M.V. Arctic trials have encompassed three different sites, ice thicknesses from 0.19 m to 1.0 m and in-ice speeds up to 6.7 m/s. They, therefore cover a fair range of ice conditions and probably provide a better than usual test of the validity of the equations of Kashtelyan, Enkvist and Poznyak. As shown in sub-section 5.2, the following tentative conclusions can be drawn from the analysis of the data:

(1) The non-velocity dependent resistance, $R_b + R_s$ varies approximately with $h^{1.8}$. Therefore the h^2 term will predominate over the h term and, since it is now generally agreed that R_b is greater than R_s, the Poznyak form of equation seems preferable.

(2) The velocity dependent resistance varies approximately linearly with V. This favours the Kashtelyan or Poznyak forms in preference to the Enkvist form for R_v.

(3) Although the data base is very limited, the indications are that the velocity dependent resistance varies with h to a power of at least 1.5. Each of the equations of Kashtelyan, Enkvist and Poznyak show a linear variation with h which does not appear to fit with the M.V. Arctic data, unless other variables such as E or σ_f are introduced into the velocity dependent term and give the same effect as an increased power of h. That this is possible is supported by Table 4.4 which shows a strong variation of E with h.

Of the second type of equation derived from dynamic or semi-dynamic considerations, the form of the Carter equation agrees well with the M.V. Arctic trials data analysis insofar as:

- the non-velocity component varies with h to a power between 2.0 and 1.25 (see equation 12)

- the velocity component varies approximately linearly with V.

- the velocity component varies with h to a power between 1.75 and 1.0 approximately.

The Carter equation agrees reasonably with the Baffin Island trials data and the Saguenay River trials data but gives a resistance much too low to suit the Lake Melville trials data unless an unreasonably high friction coefficient is assumed. The reason for the low resisitance indicated for Lake Melville is partly due to the unusally high E/σ_f ratio measured for the Lake Melville ice and the inclusion in the Carter equation of the ratio $\sigma_f/E^{0.25}$. The effect of the ratio $\sigma_f/E^{0.25}$ for Lake Melville ice compared with Baffin Island ice measurements is shown below:

	Baffin Island				
	Site 2	Site 3	Site 4	Site 5	Lake Melville
σ_f kPa	350	340	330	410	267
E MPa	525	1090	1320	1400	2320
$\sigma_f/E^{0.25}$	13.1	10.5	9.73	11.9	6.83

If the values of σ_f = 380 kPa and E = 5000 σ_f used by Carter are substitued for the measured Lake Melville values, $\sigma_f/E^{0.25}$ becomes 10.23 instead of 6.83 and the quasi-static breaking component of resistance would be increased by 48% which agrees well with the measured resistance. The inference is that, either the Carter equation does not accurately reflect the effect of σ_f and E on the ice resistance, or that the measured values of σ_f and E do not always represent the actual values under icebreaking conditions. The latter alternative is quite possible insofar as the length of the tested cantilevers may be quite different from the bending length of the broken ice pieces and the loading time may also be very different. This should be a matter for further study.

The Mathews equation is based on entirely dynamic considerations. It also includes the ratio of $\sigma_f/E^{0.25}$ in the first term. However, in the second, velocity dependent term, resistance is dependent on $h^{1.5}$, $V^{0.5}$ and $E^{0.25}$ whereas in the Carter equation the velocity resistance is dependent on a combination of $h^{1.75}$, h, V, $\sigma_f^{0.5}$ and $E^{0.25}$ approximately.

The equations of Carter and Mathews cannot readily be broken down into the three components of resistance postulated by Kashtelyan. The Carter equation can, however, be broken down into two main components, of which one is dependent on velocity and the other is not. The Mathews equation can be broken down into two components of which one has $V^{0.5}$ in the numerator and the other has $V^{0.5}$ in the denominator.

The components of resistance given by the various equations are compared in Tables 5.3 and 5.4 for Lake Melville ice with σ_f = 267 kPa, E = 2320 MPa, h = 0.61 m and V = 2.0 m/s and with two different values of f_d = 0.03 and 0.13.

Tables 5.3 and 5.4 clearly show the inexactness of the present science of predicting the resistance of level ice. However, recent analytical studies, full-scale test and model experiments have given a better understanding of the size of the various components of resistance and some of the above equations could almost certainly be modified so as to improve their predicting capability.

For instance, recent full-scale tests reported by Enkvist[28] have shown that, of the non-velocity dependent

TABLE 5.3
Resistance for $f_d = 0.03$

EQUATION	RESISTANCE (kN)				
	R_b	R_s	R_V	R_{ow}	R_{Total}
Modified Kashtelyan	23	438	194	38	693
Modified Poznyak & Ionov	331	103	230	38	702
Enkvist (2)	6	174 (323)	225 (76)	38	443
Carter	310	0	140	38	488
Mathews (1)	591		277	38	906

TABLE 5.4
Resistance for $f_d = 0.13$

EQUATION	RESISTANCE (kN)				
	R_b	R_s	R_V	R_{ow}	R_{Total}
Modified Kashtelyan (1)	26	488	256	38	808
Modified Poznyak & Ionov	574	138	290	38	1040
Enkvist (2)	8	194 (408)	323 (109)	38	563
Carter	458	0	207	38	703
Mathews	837		764	38	1639

NOTE: (1) Value shown under $R_b + R_s$ is dependent on $1/V^{0.5}$

(2) Enkvist has a term in his expression for velocity resistance which is not actually dependent on velocity. The figures in brackets for R_s and R_V adjust for this.

resistance, 40 to 80 percent is due to breaking forces and 60 to 20 percent due to inertial or submersion forces. Enkvist gives a new equation for the submersion component (see Appendix A, page A-17) and, applying this to the M.V. Arctic, the following values of R_s are obtained in kiloNewtons for h = 0.61 m and f_d as indicated:

f_d	=	0.03	0.08	0.13
R_s	=	68	109	136

As pointed out in sub-section 5.4.3, Poznyak and Ionov have shown from model tests that, for a particular ice thickness and $f_d = 0.08$ a reasonable ratio for breaking/submersion resistance would be 72/28, making the breaking resistance 2.6 times the submersion resistance. Unfortunately, most equations have the breaking resistance varying with a different power of h from the submersion resistance thus making a constant ratio impossible. However, there is strong evidence that the breaking resistance in the Kashtelyan and Enkvist equations should be greatly increased with some reduction of R_s.

In general, it may be said that the various analytical equations require detailed examination to determine the reasons for the various inconsistencies, as a basis for putting forward an equation which better fits the recent full-scale and model data. Such a task would have to be the subject of a special study.

In the meantime, pending further study, the following tentative conclusions can be drawn.

1. The modified Kashtelyan equation embodying friction factors related to the Shimansky form coefficients fits the trials data fairly well but has a ratio of R_b/R_s which is now known to be far too small.

2. The Enkvist equation, while embodying all the necessary parameters, badly underestimates the non-velocity resistance, particularly the breaking component.

3. The Carter equation embodies an attractive new analytical approach but some assumptions need further examination. Its dependence on σ_f and E does not appear to fit the trials data but the fault could be that the measured values of σ_f and E do not represent the true values under actual ice breaking conditions.

4. The Mathews equation is new and needs testing against a variety of ships and ice conditions.

Of the equations tested, it appears that the modified Poznyak and Ionov equation (Eqn. 10) best fits the observed data if the assumption of f_d equals about 0.03 for the trials conditions can be accepted. It is recommended that this equation be tested further along with others against other ships and against the data from the model test recently undertaken in Hamburg on different bow shapes for the M.V. Arctic.

5.6 Performance in Broken Channels and Pack Ice

The treatment of vessel resistance in broken ice is a complex problem related to the actual interaction between individual floes and the hull itself.

Kashtelyan (1968)[1] developed an empirical expression describing vessel resistance in broken ice over 6/10 concentration involving four types of forces:

- Inertia forces in vicinity of the stem
- Hydrodynamic resistance to motion of the ice elements
- Friction forces between individual ice elements
- Elastic forces of compression between elements (so-called static components, not velocity dependent).

Kashtelyan's expression has been run using a micro computer, for the M.V. Arctic, using a number of different ice size parameters considered representative of the 1981 voyage records.

The following values were input as constants.

mass density of sea ice - taken as	900 kg/m^3
hull/ice friction coefficient (assumed a 30% reduction due to Inerta 160)	0.07
Ship Length	209.6 m
Ship Beam	22.9 m
Ice Waterplane Coefficient	0.85
Bow Waterplance Coefficient	0.70
Bow Entering Angle	32.8°

Variables are:

Ice Floe Dimension	1 - 12 m
Ice Floe Thickness	20 - 60 m

Channel Width/Vessel Beam

- Used Ratio of 10 in Naturally Broken Ice
- Ratio of 1.5 - 1.8 when following Louis St. Laurent in 20 to 60 cm first-year ice.

Velocity	2 - 16 Kn.
Ice Packing	4 - 10/10

Predicted resistance was compared with 1981 voyage records showing ice concentrations of 9+/10 or less in naturally broken ice or up to 10/10 when following the Louis St. Laurent. Table 4.14 shows the range in MV Arctic ice resistance determined for all cases defined as broken ice in 1981, along with associated ship speed and ice conditions. Broken ice resistance is critically dependent on the floe size parameter rh (dimensions and thickness), but, there are no records of actual floe dimensions in the MV Arctic data package. In order to compare full scale broken ice resistance with predictions, Kashtelyan's formulation has been plotted for a reasonable assumed range of ice and channel parameters. Results are shown in Figures 5.19 to 5.21.

Figure 5.19 shows ice resistance in naturally broken ice for concentrations 4 to 9+/10. Curves have been plotted for 3, 9 and 15 knots. Ice thickness was assumed as 25 cm, which was the average first-year ice thickness encountered in 1981. Floe sizes varied according to packing, from 4 m at 4/10 toll m at 9+/10. Actual MV Arctic resistances have been superimposed on this graph with the range in velocities noted. There is tremendous scatter in the full-scale data with no distinct speed dependence. In general, the MV Arctic data does show increasing resistance with packing. Kashtelyan's predictions compare favourably in order of magnitude with actual MV Arctic values.

Figures 5.20 and 5.21 show the predicted variation in MV Arctic resistance with velocity, following an icebreaker in ice of 10/10 packing. Figure 5.20 plots resistance at three different ice thickness values with a constant rh factor of 5, i.e., ice floe size varying from 8.8 m in 60 cm of ice, to 25 m in 20 cm of ice.

Figure 5.21 shows the variation of resistance with velocity at three values of rh. The ratio of channel width left by the icebreaker to MV Arctic beam, ranged from 1.5 to 1.8. Ice thickness was kept constant at 40 cm, representative of the MV Arctic following the Louis St. Laurent in November 1981. Actual MV Arctic data is plotted for comparison with predicted curves. Figures 5.20 and 5.21 demonstrate the sensitivity of Kashtelyan's predictive formulations to ice floe size in the channel. Variations in ice packing can also have an even more dramatic effect on resistance, as shown in Figure 5.19.

Broken ice resistance in floes from 2 to 20 m in diameter is critically dependent on:

- vessel beam
- channel width
- concentration or packing in the channel
- ice floe size
- ice floe thickness
- friction coefficient

Although Kashtelyan's equations for finely broken ice resistance takes into account bow shape details such as waterplane coefficient and stem angle, these ship parameters have much less effect on total resistance than vessel beam or ice conditions.

Packing is probably the most important single parameter. At high concentrations (>7/10), an increase of less than 2/10 in packing can easily double the total resistance. Below about 6/10 concentrations there is minimal ice resistance. A relatively low number of floes participate in the ice/vessel interaction process, and there is a distinct lack of packed ice ahead of the stem. At very high packing values, individual floes can become in a state of compression. There is then a major redistribution of the magnitude of different broken ice resistance terms, accompanied by sharp rise in the static component. Ship velocity and floe dimensions become less important once ice in the channel is in a state of compression.

At speeds less than 3 knots, velocity dependent terms can be neglected, as broken ice resistance reduces to a static component with an error of less than 10%.

In the case of a naturally occurring broken ice field with large floes, a much simpler predictor equation is considered adequate, given the extreme potential variations in ice encounter as a function of time. Kashtelyan uses the relationship:

$$R = (C_k + 9V^{1.3})\ h^{1.2}$$

where h = ice thickness (0.8 to 2 m)

V = average transit velocity (0.5 to 13 knots)

C_k = empirical coefficient based on floe size (20 m to 1.5 km)

This prediction presents a time averaged resistance which must become increasingly coarse as floe size increases and concentration decreases (short bursts of continuous icebreaking interspersed with open water).

Kashtelyan postulates a critical floe dimension above which ice resistance can be considered independent of floe size (e.g. >1,100 m for speeds of 2 knots; >600 m at 10 knots).

In summary, Kashtelyan's empirical predictions appear to provide realistic estimates of MV Arctic resistance in broken ice. Further interpretation and refinement of broken ice predictor equations will require a complete description of the broken ice environment, something lacking in existing voyage records or test logs.

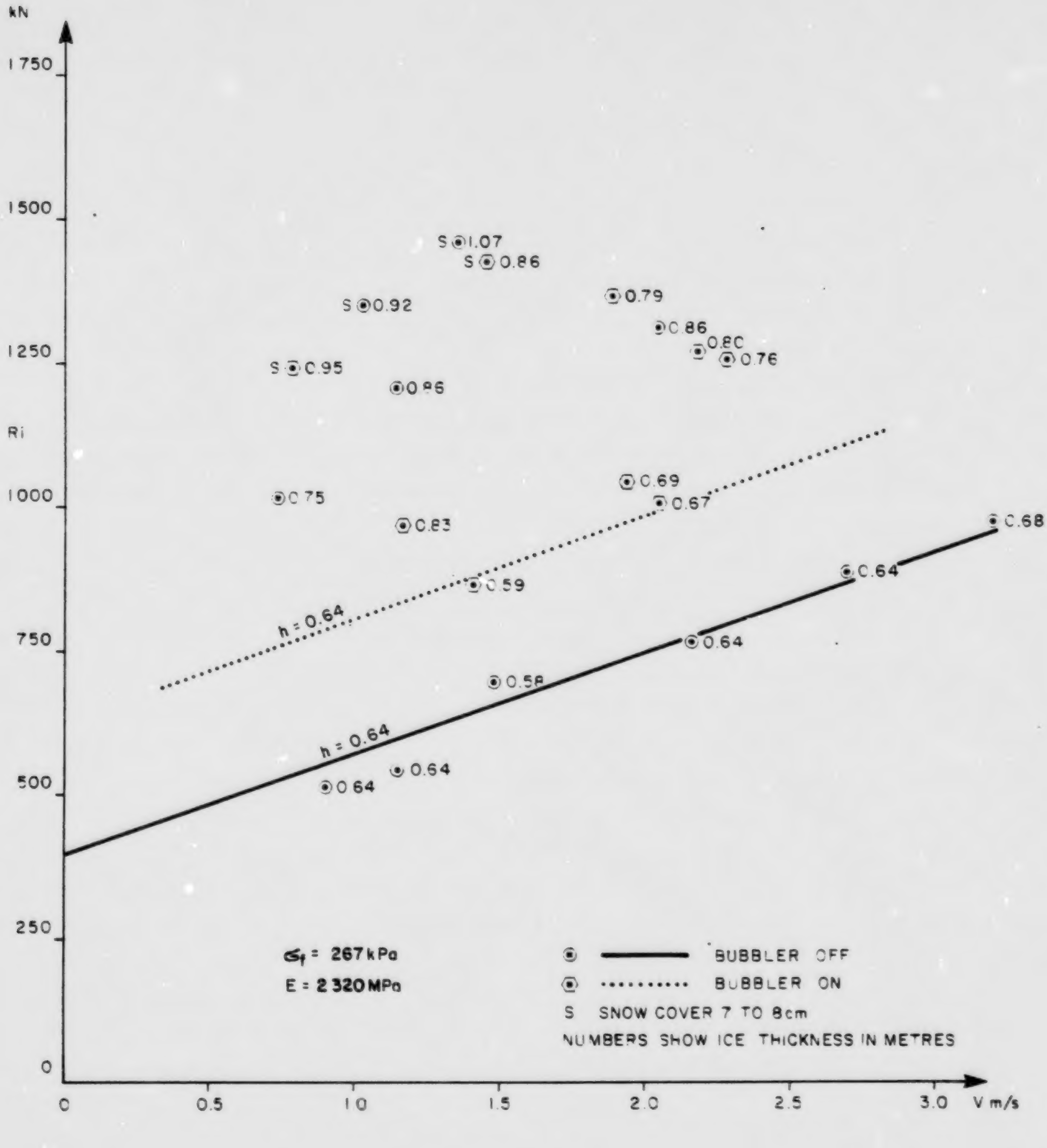


FIGURE 5.1 1981 LAKE MELVILLE TRIALS. ICE RESISTANCE VS. SPEED

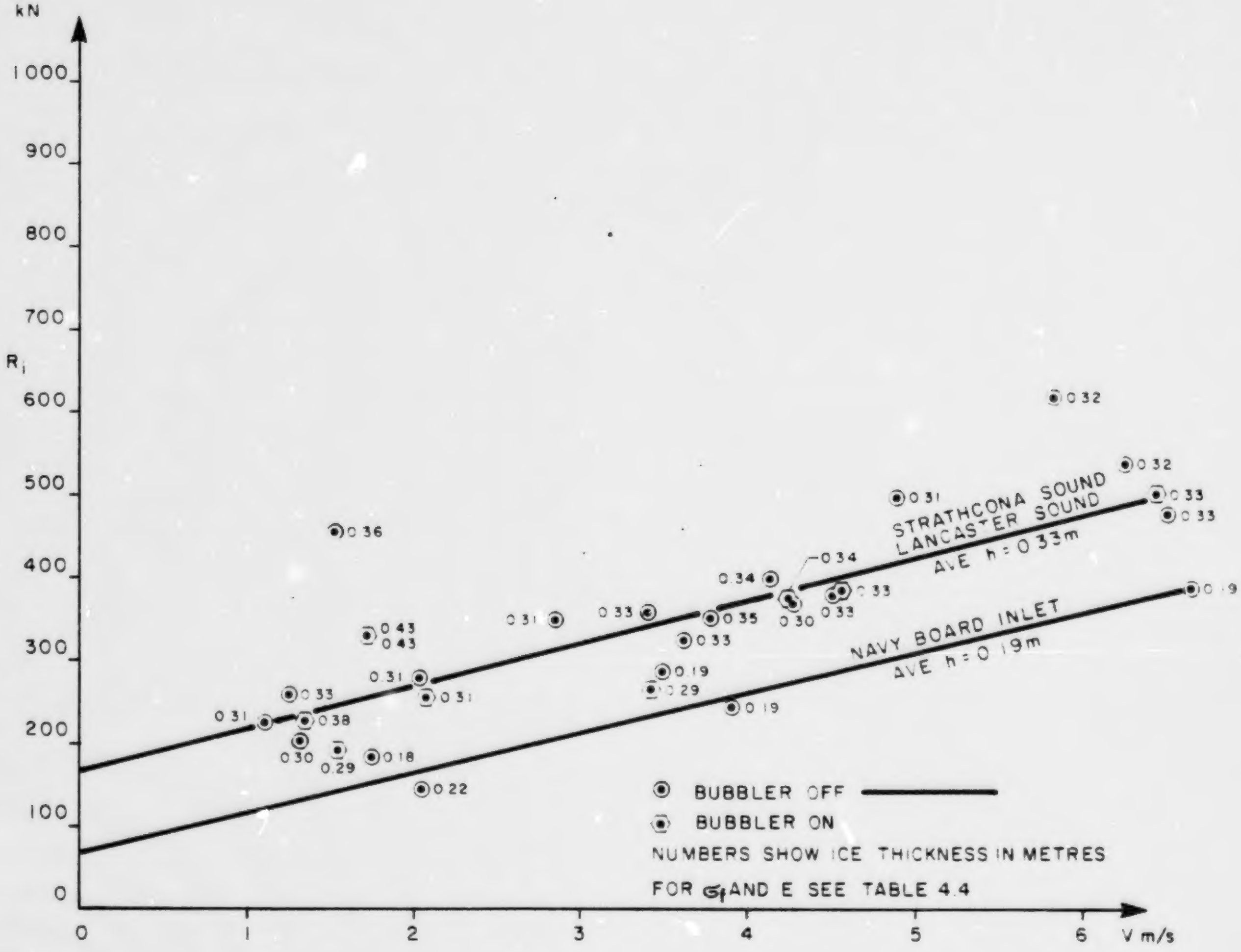


FIGURE 5.2 1980 BAFFIN ISLAND TRIALS. ICE RESISTANCE VS. SPEED

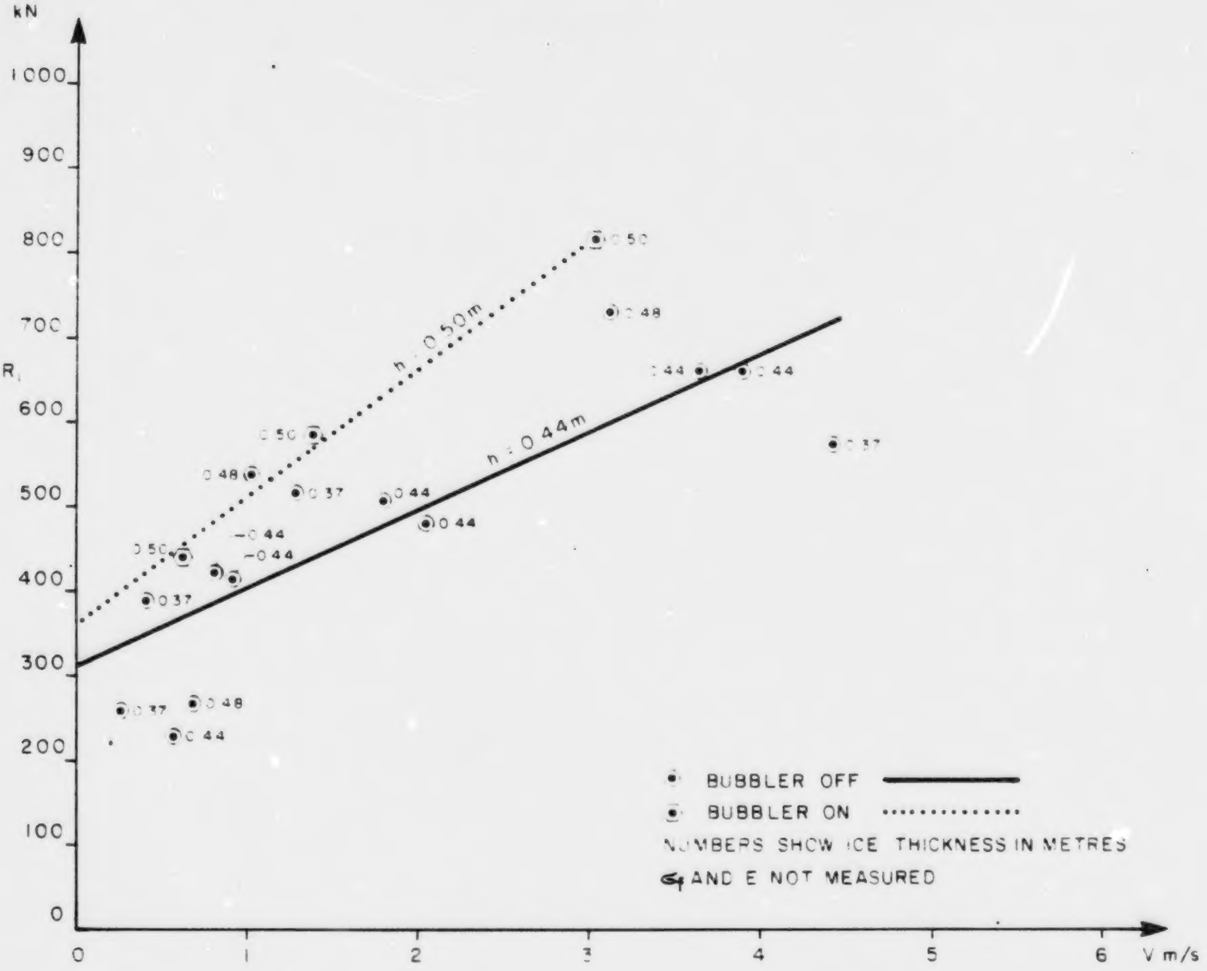


FIGURE 5.3 1979/80 SAGUENAY RIVER TRIALS. ICE RESISTANCE VS. SPEED

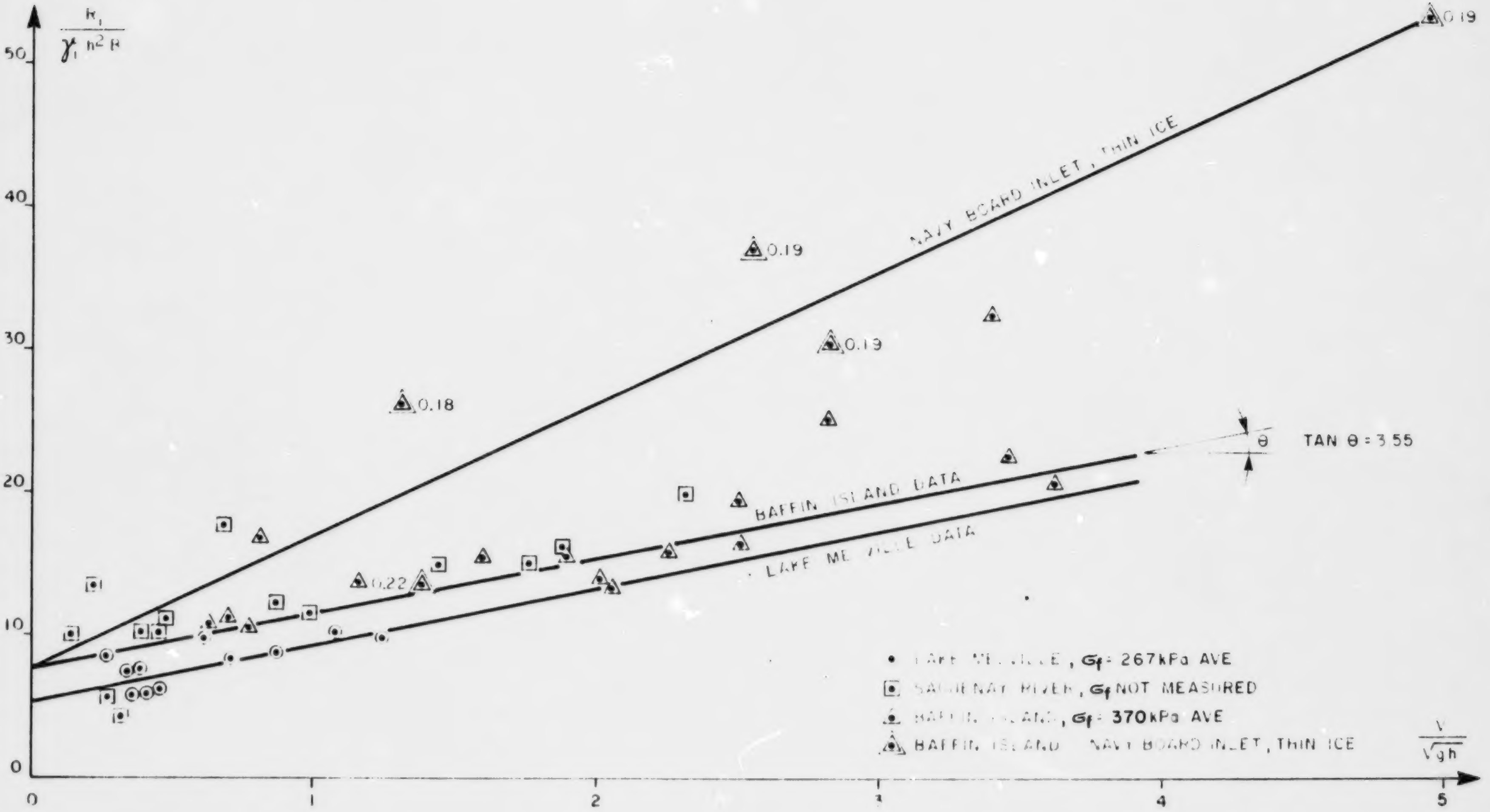


FIGURE 5.4 DIMENSIONLESS PLOT $R_i/\gamma_i\ h^2\ B$ VS. $V/\sqrt{gh}$

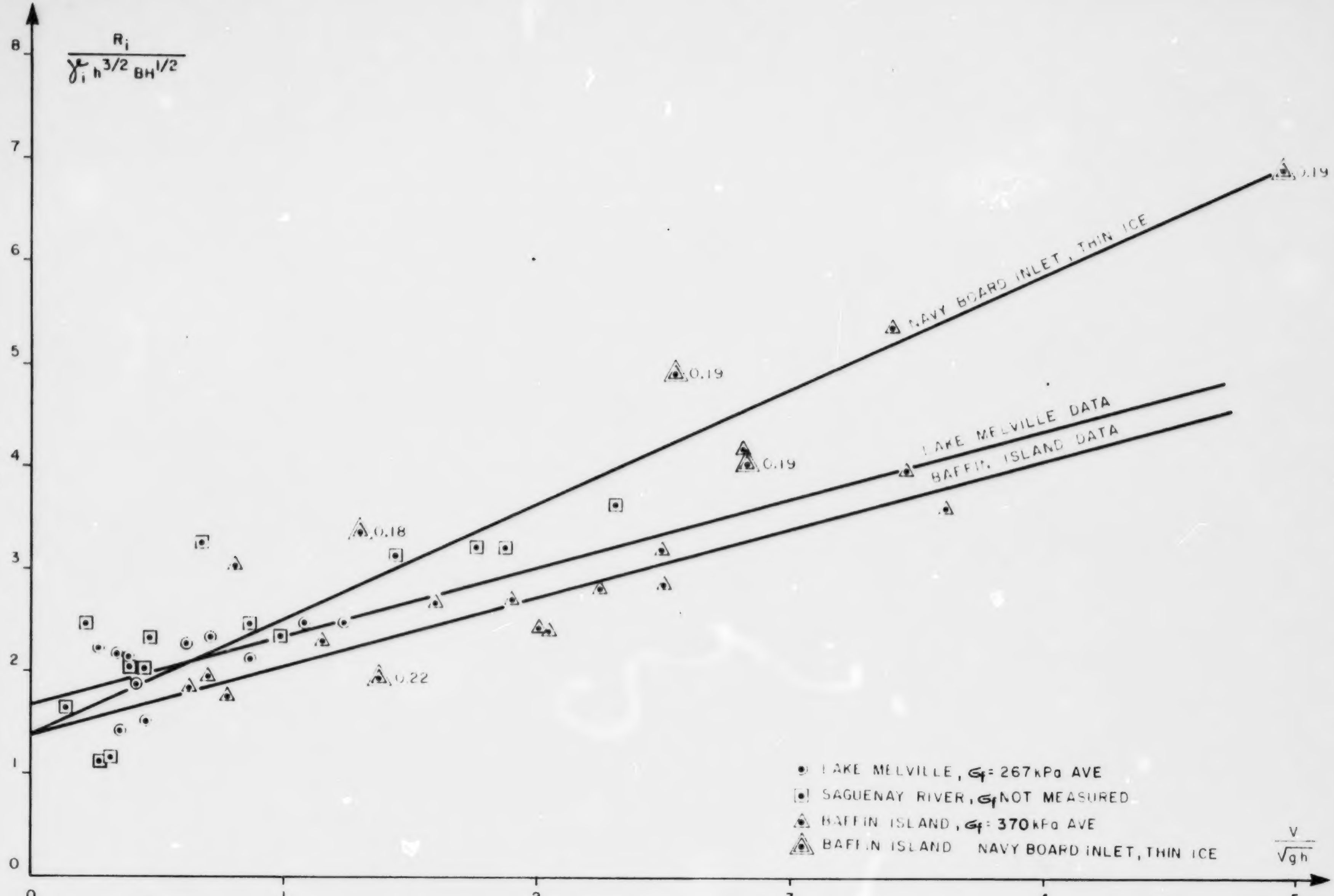


FIGURE 5.5 DIMENSIONLESS PLOT $R_i/\gamma_i h^{1.5} BH^{0.5}$ VS. $V/\sqrt{gh}$

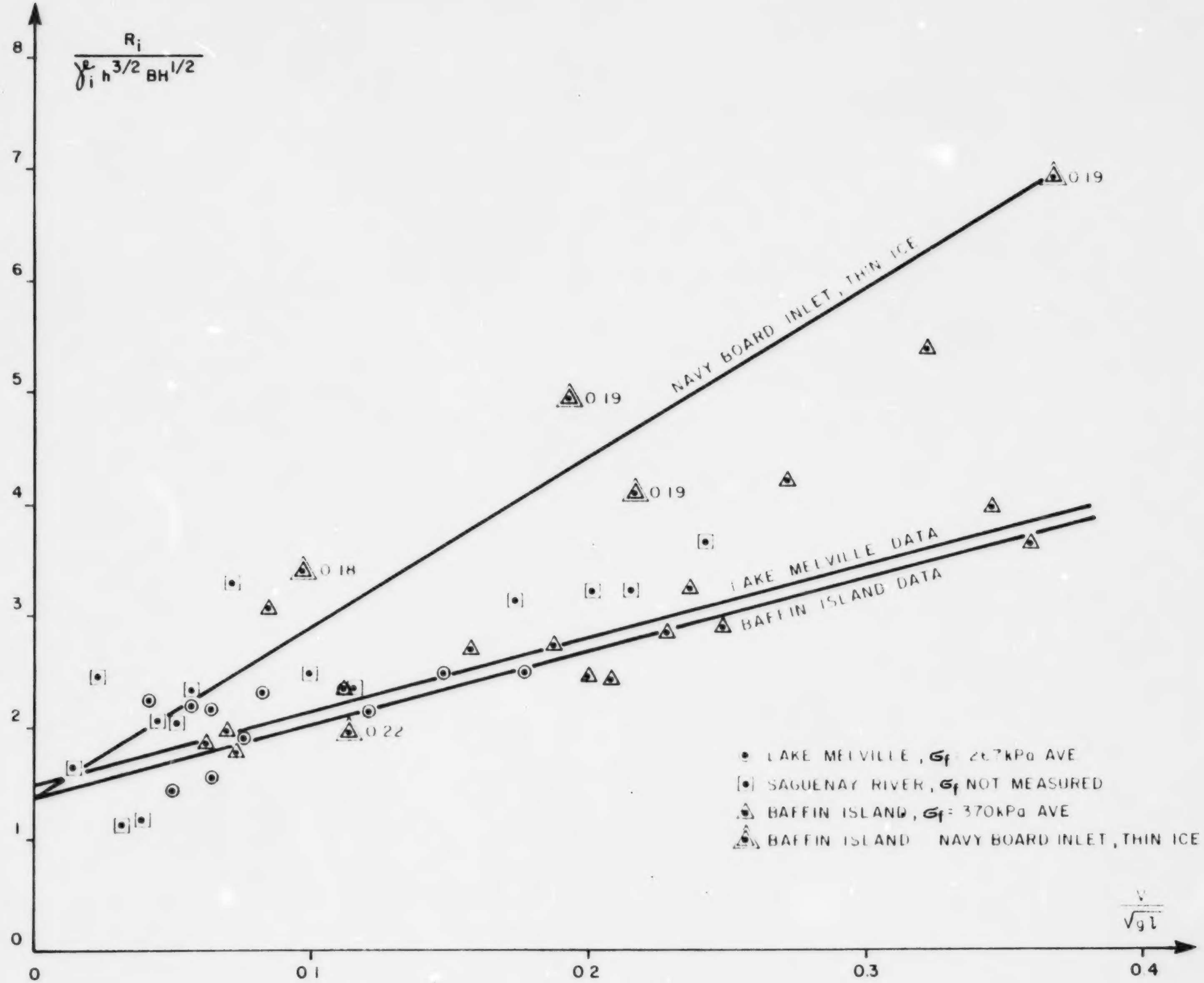


FIGURE 5.6 DIMENSIONLESS PLOT $R_i/\gamma_i\ h^{1.5}\ BH^{0.5}$ VS. $V/\sqrt{gL_w}$

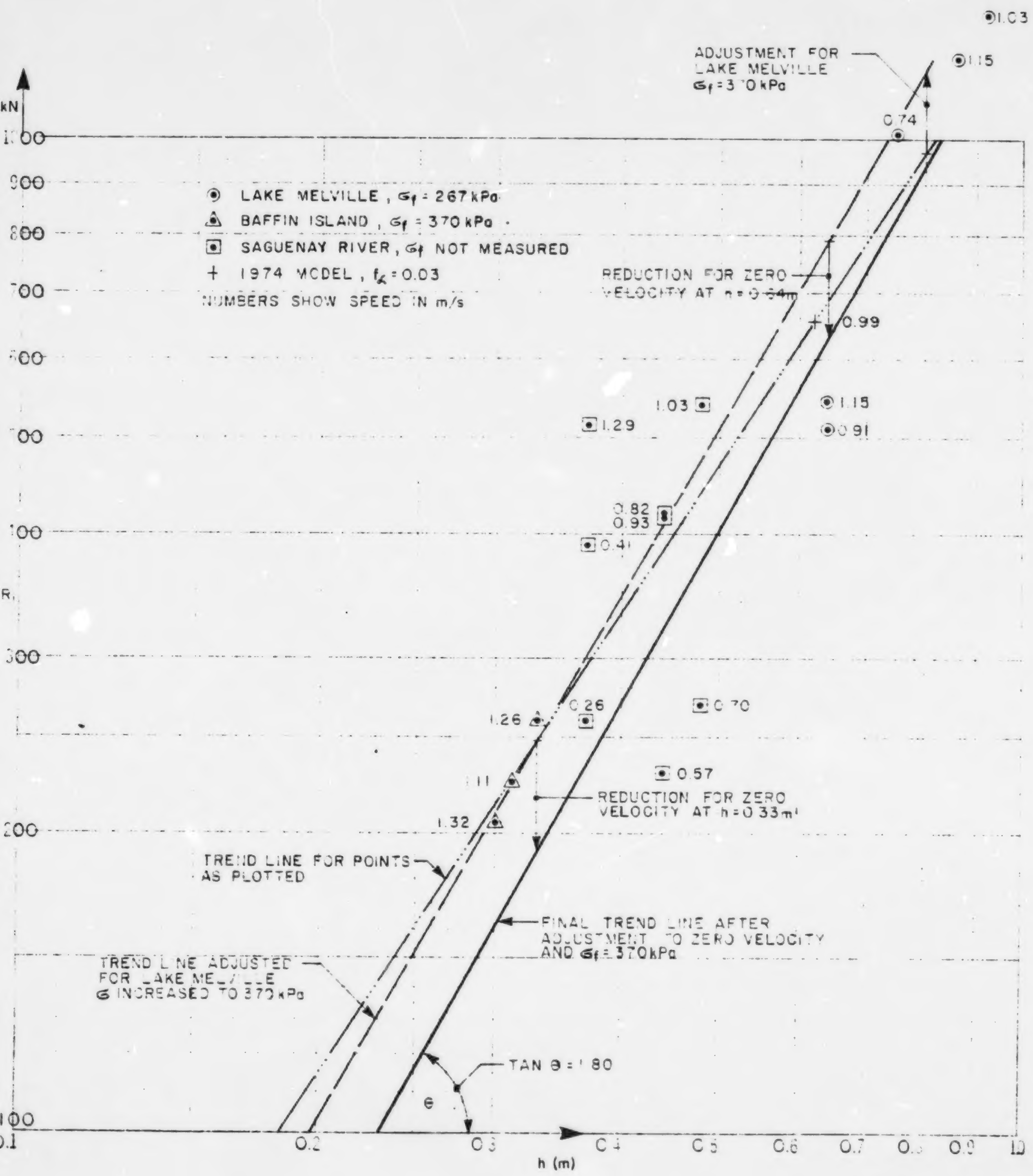


FIGURE 5.7 LOGARITHMIC PLOT, ICE RESISTANCE VS. ICE THICKNESS FOR SMALL VELOCITIES

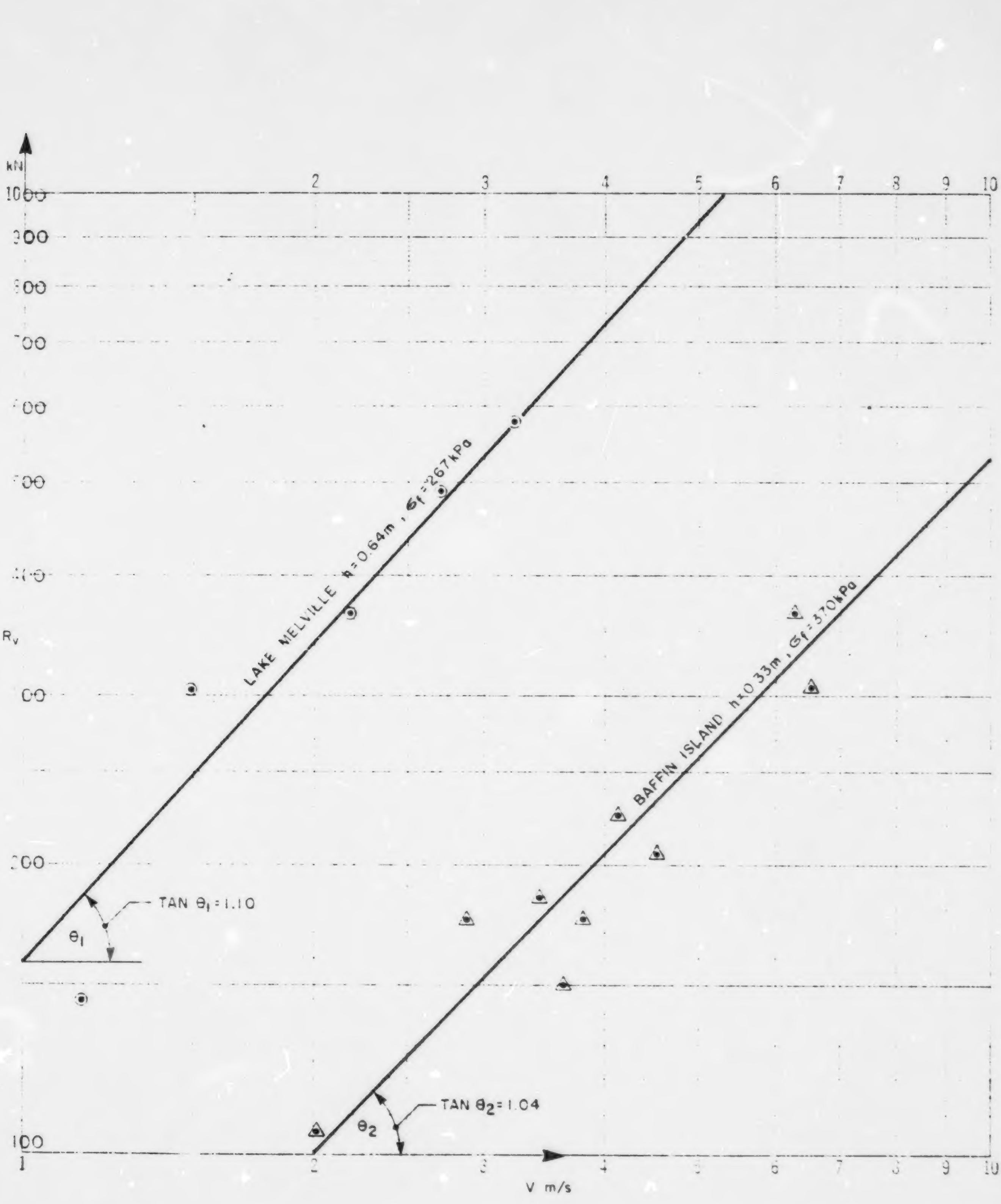


FIGURE 5.8 LOGARITHMIC PLOT, VELOCITY RESISTANCE VS. VELOCITY

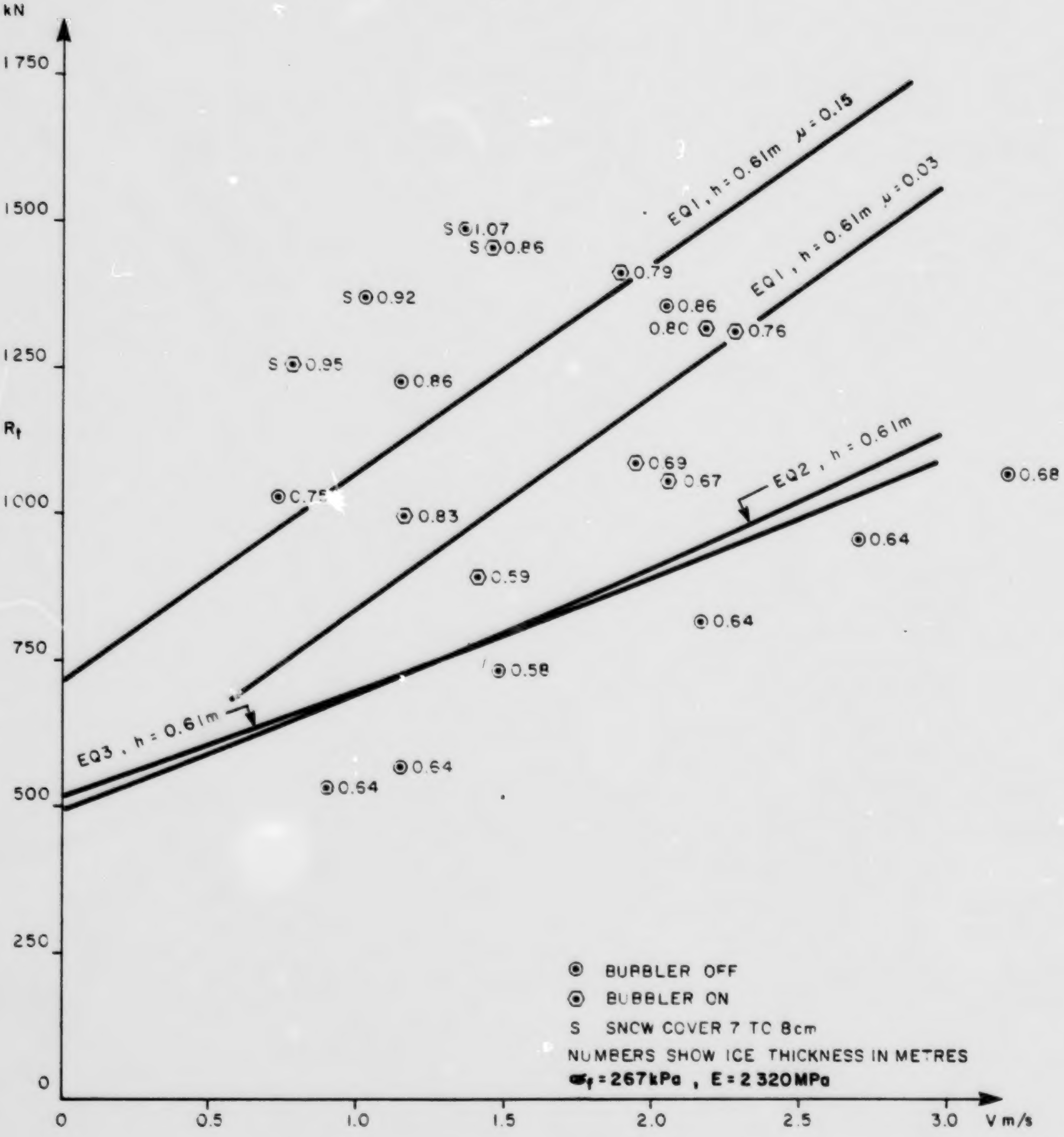


FIGURE 5.9 1981 LAKE MELVILLE TRIALS. COMPARISON WITH EXISTING PREDICTOR EQUATIONS GIVEN IN TABLE 5-1

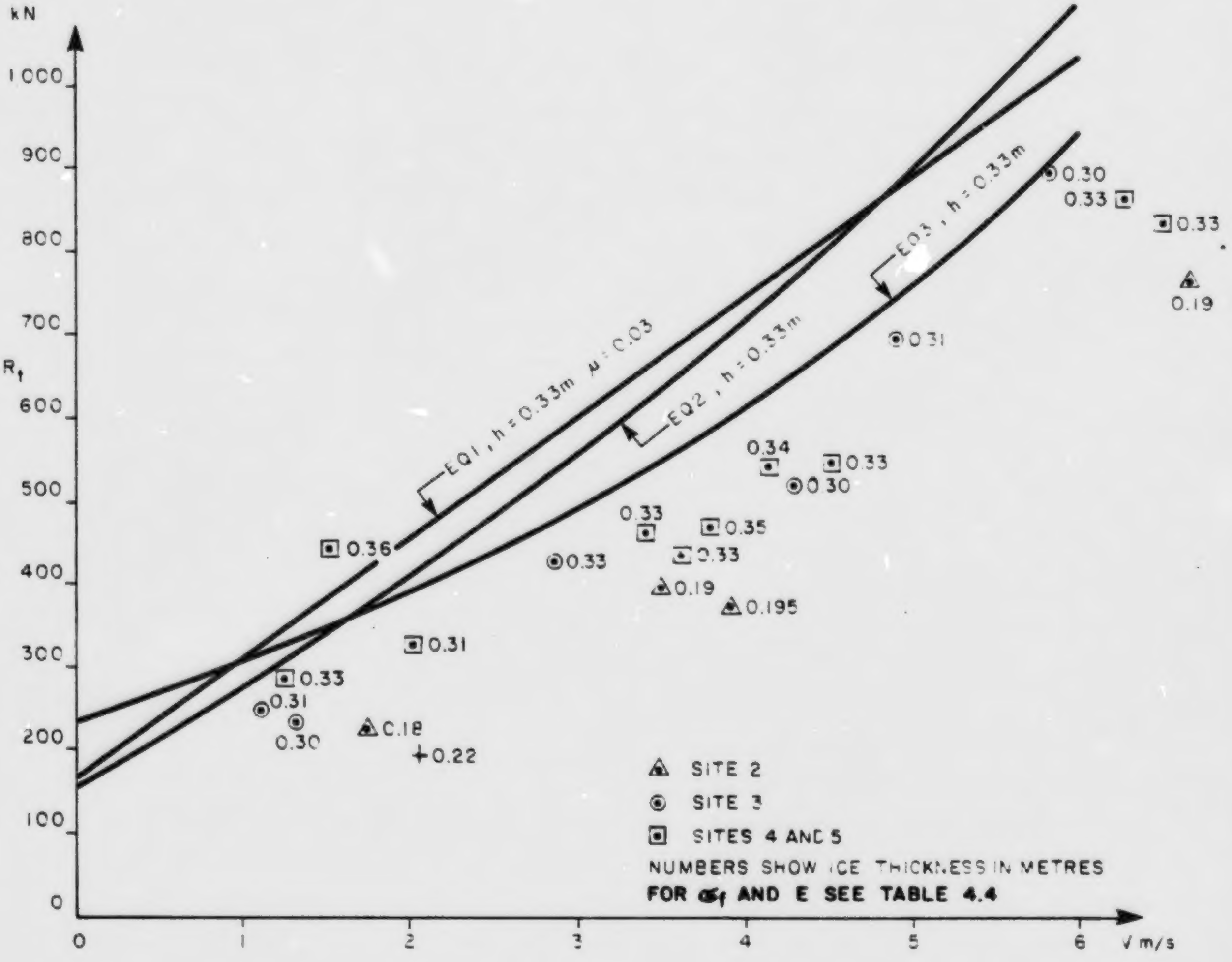


FIGURE 5.10 1980 BAFFIN ISLAND TRIALS. COMPARISON WITH EXISTING PREDICTOR EQUATIONS GIVEN IN TABLE 5-1

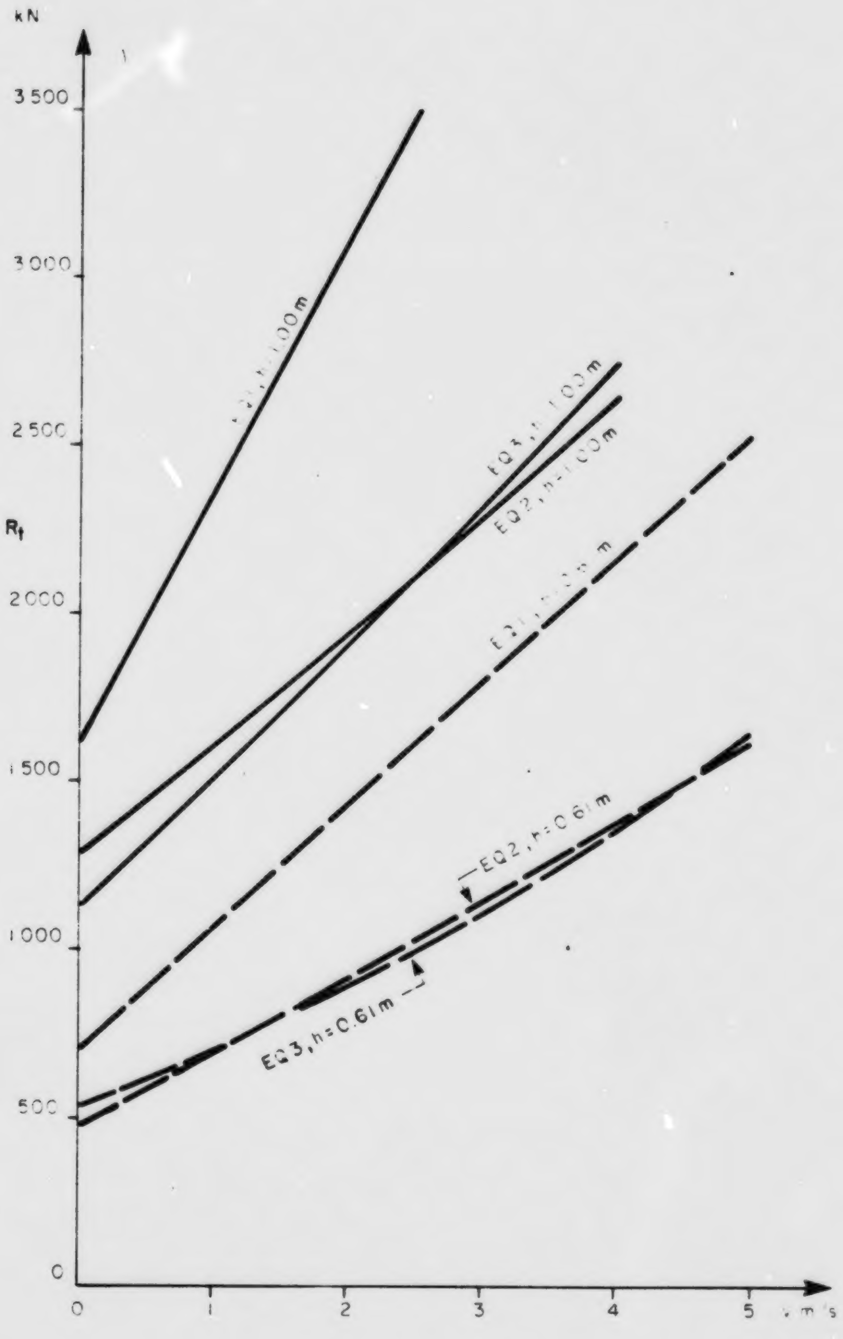


FIGURE 5.11 COMPARISON OF PREDICTOR EQUATIONS IN TABLE 5-1
RESISTANCE VS. SPEED

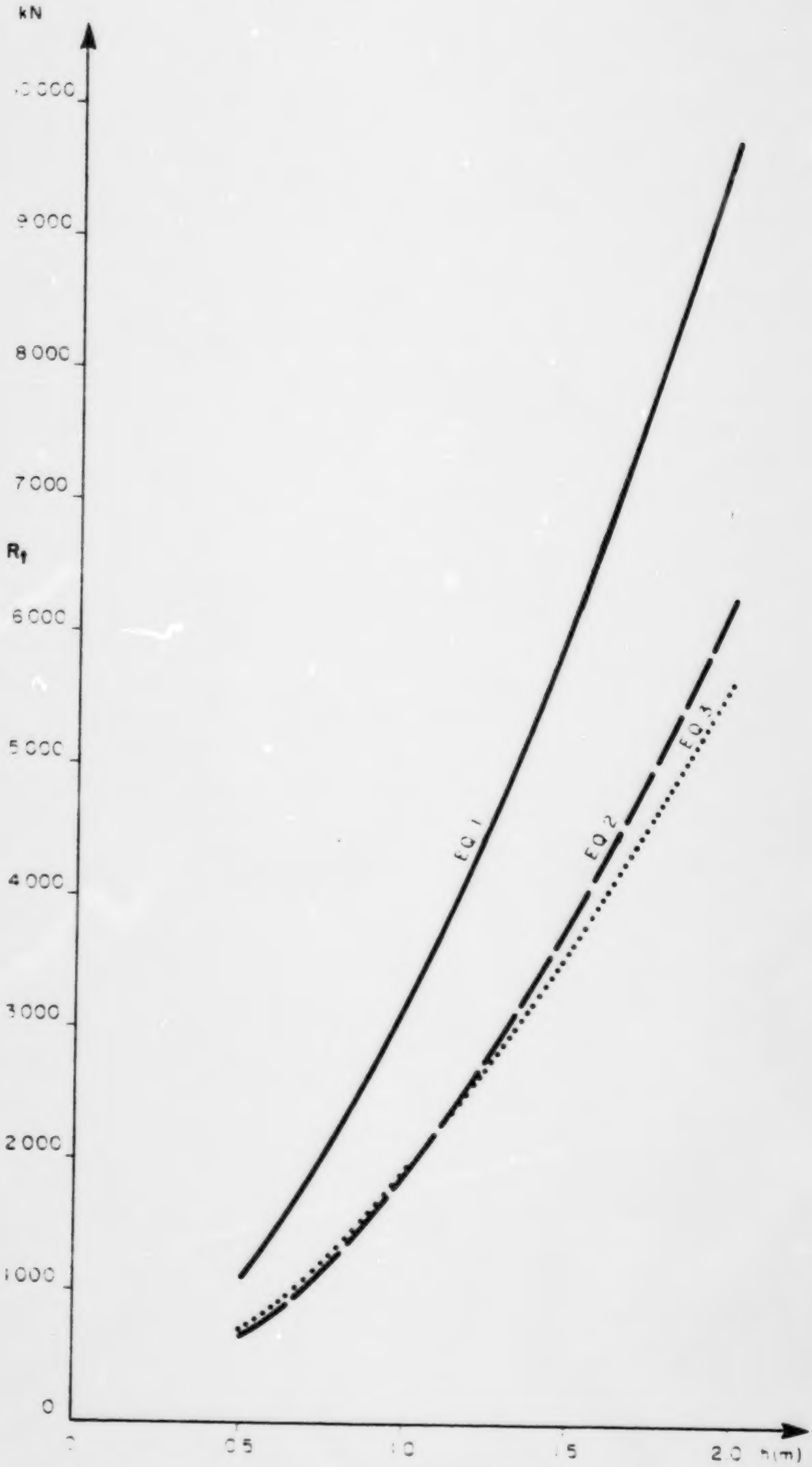


FIGURE 5.12 COMPARISON OF PREDICTOR EQUATIONS IN TABLE 5-1 RESISTANCE VS. ICE THICKNESS

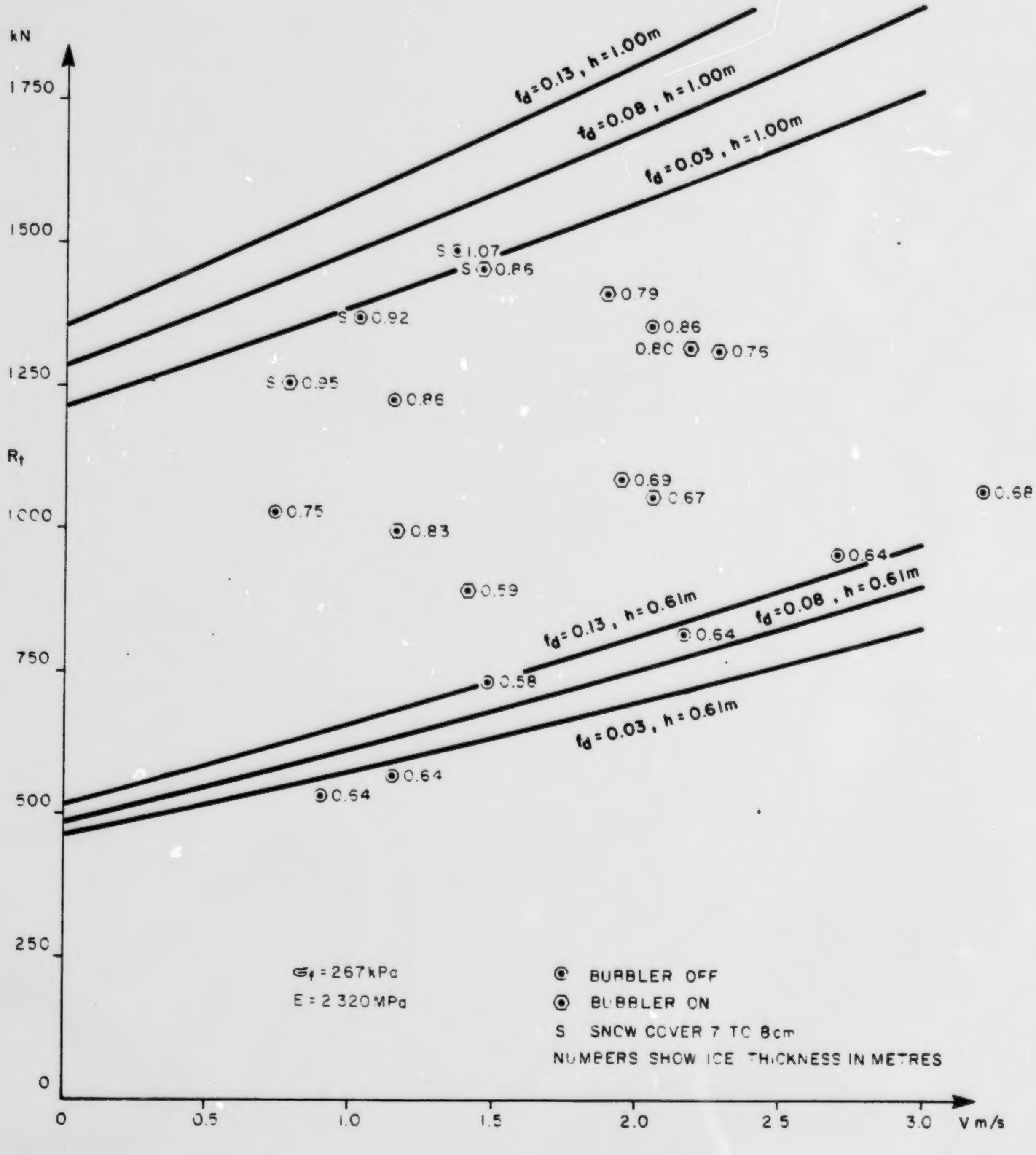


FIGURE 5.13 MODIFIED KASHTELYAN EQUATION INCORPORATING FRICTION COEFFICIENTS COMPARED WITH LAKE MELVILLE TRIALS DATA

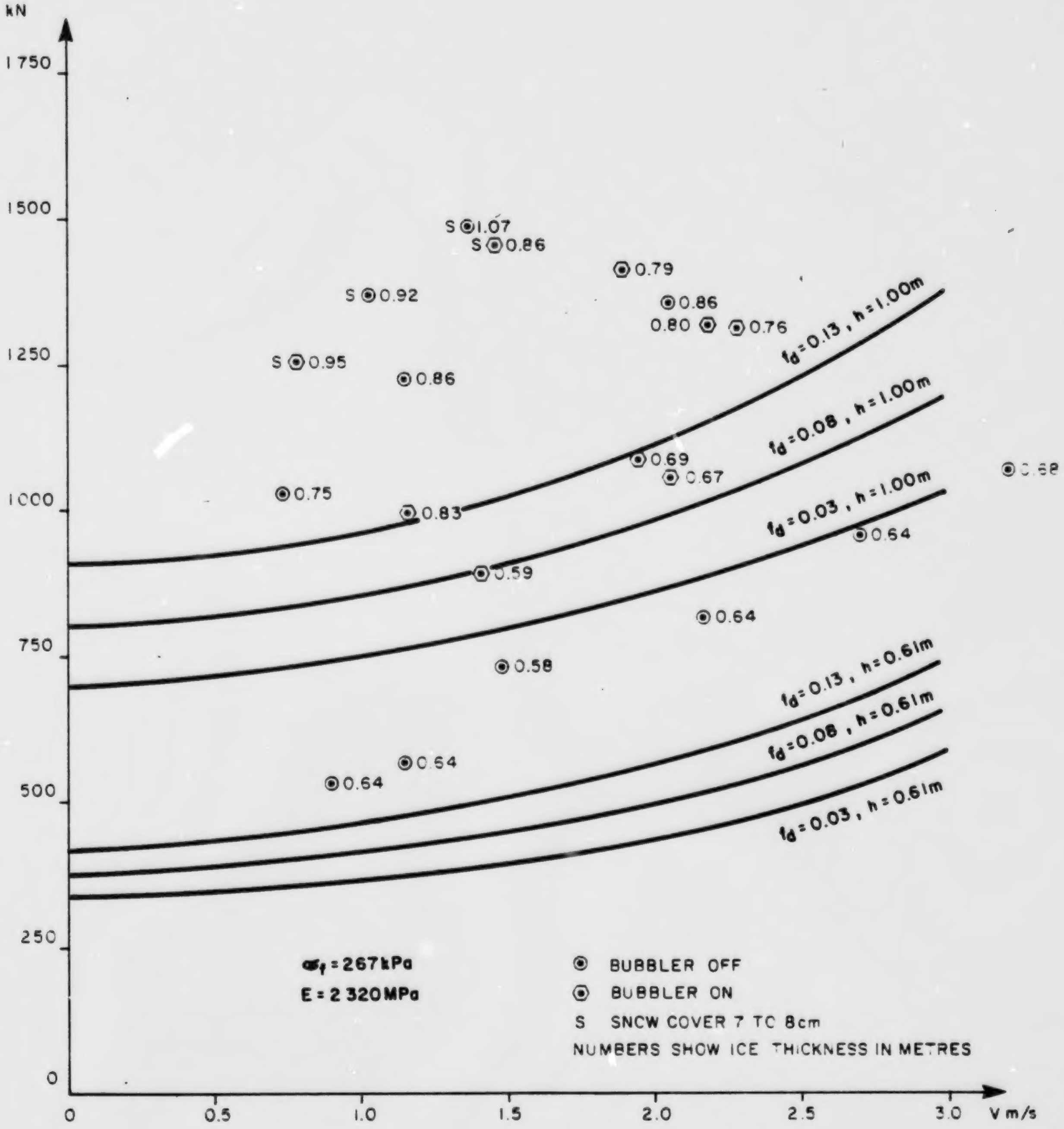


FIGURE 5.14 ENKVIST EQUATION COMPARED WITH LAKE MELVILLE TRIALS DATA

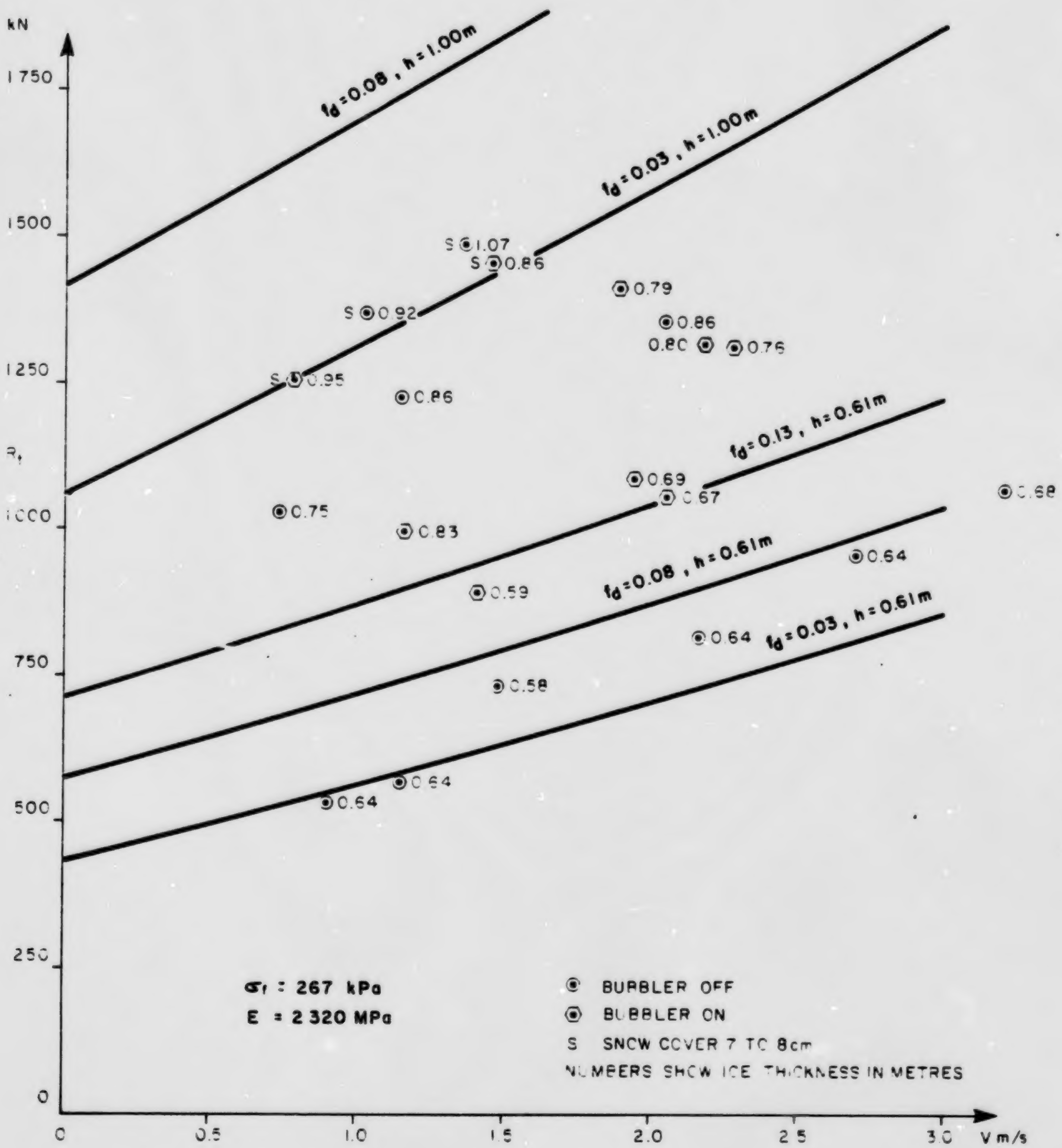


FIGURE 5.15 MODIFIED POZNYAK AND IONOV EQUATION COMPARED WITH LAKE MELVILLE TRIALS DATA

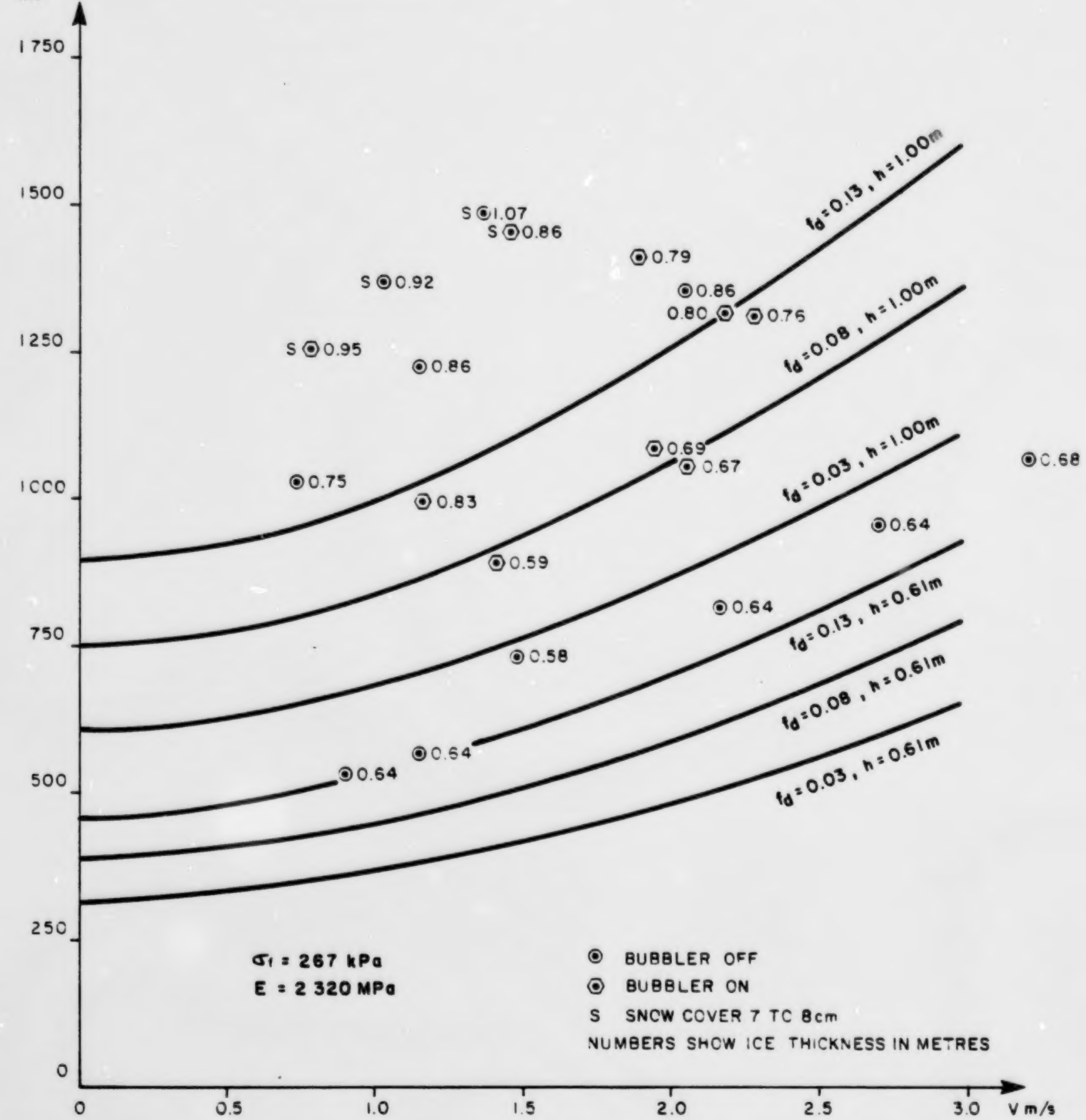


FIGURE 5.16 CARTER EQUATION COMPARED WITH LAKE MELVILLE TRIALS DATA

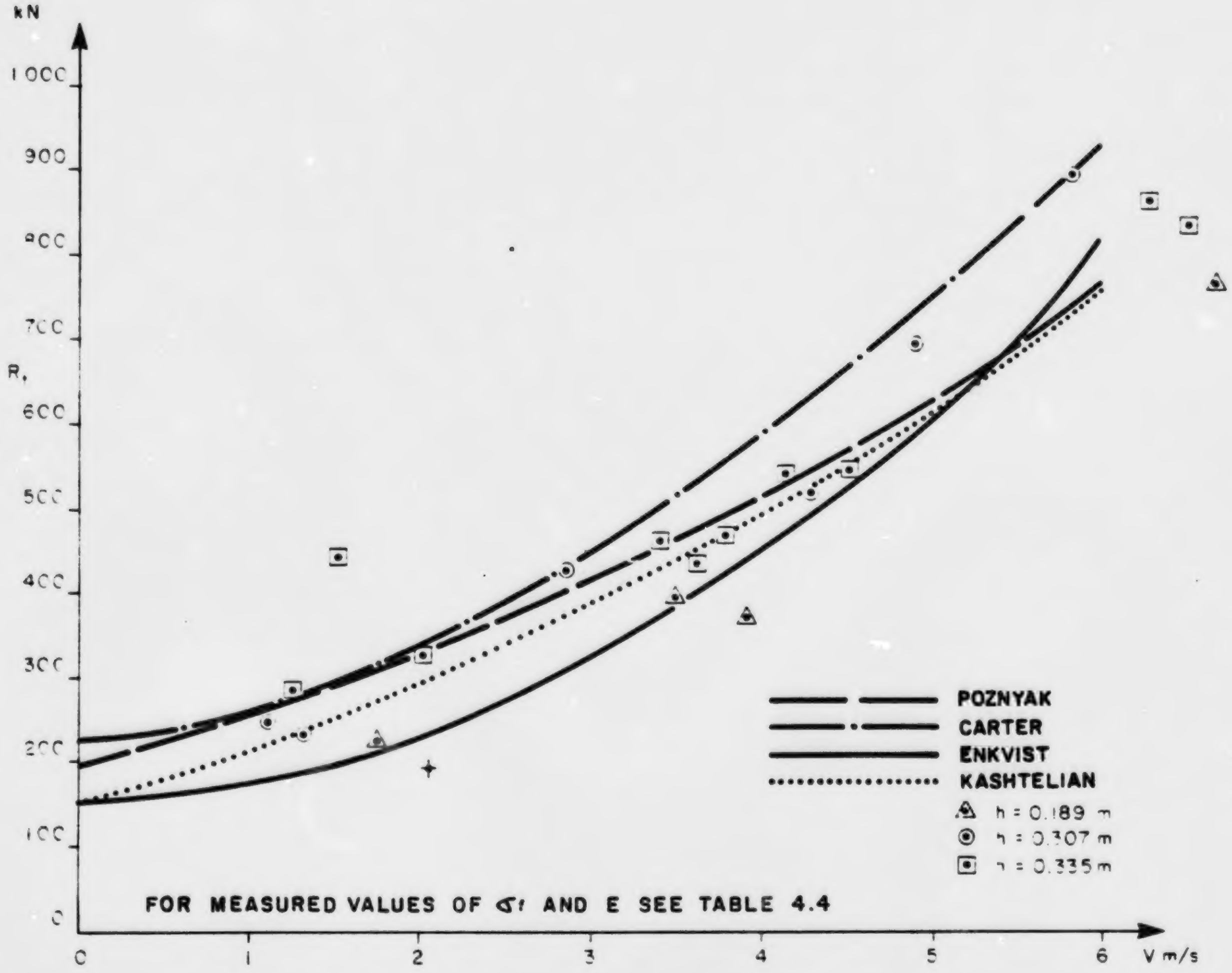


FIGURE 5.17 SELECTED EQUATIONS COMPARED WITH BAFFIN ISLAND TRIALS DATA; $f_d = 0.03$, $h = 0.335$ m, $\sigma_f = 370$ kPa, $E = 1360$ MPa

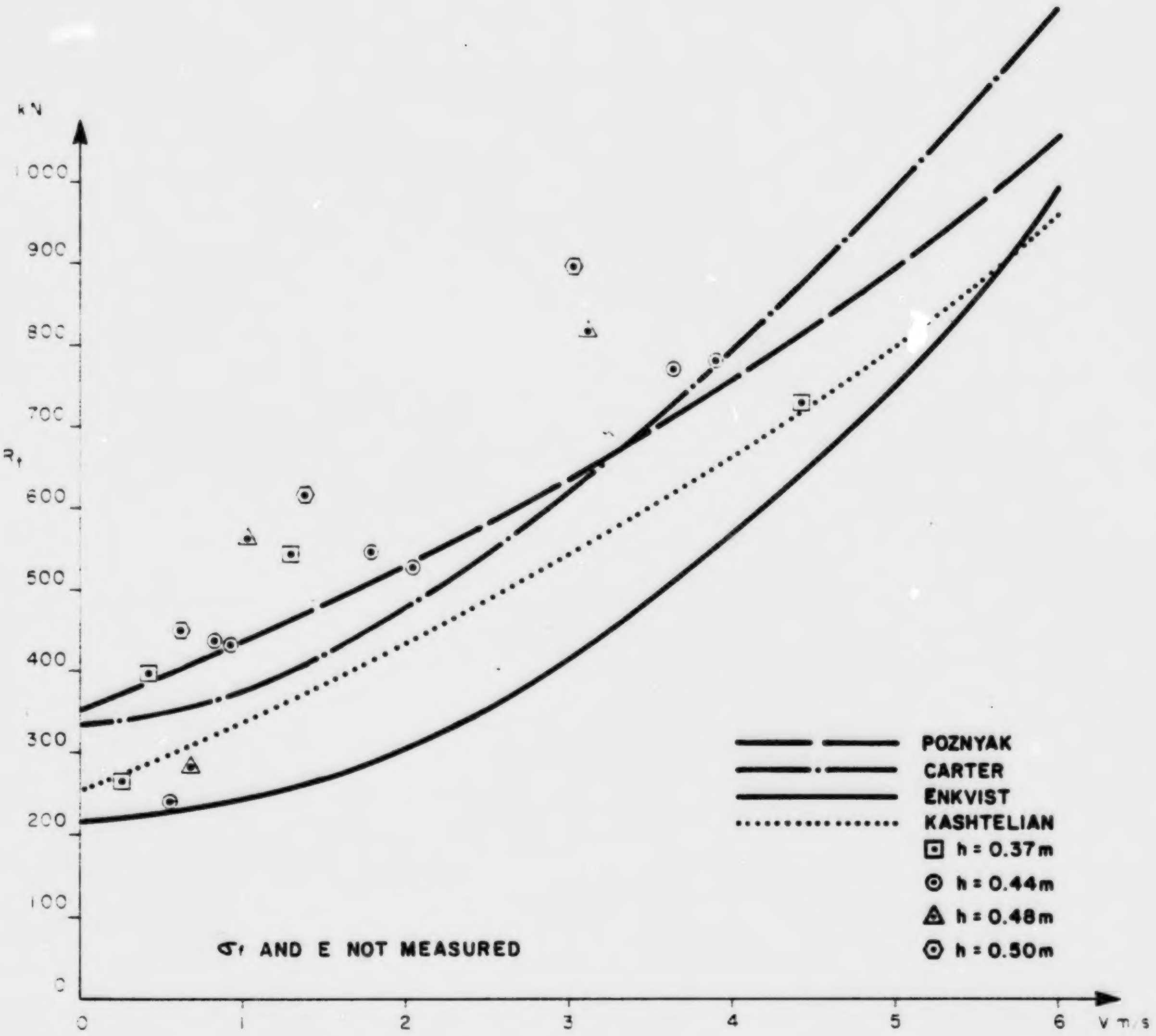


FIGURE 5.18 SELECTED EQUATIONS COMPARED WITH SAGUENAY RIVER TRIALS DATA; f_d = 0.03, h = 0.44 m, σ_f = 425 kPa, E 1875 MPa

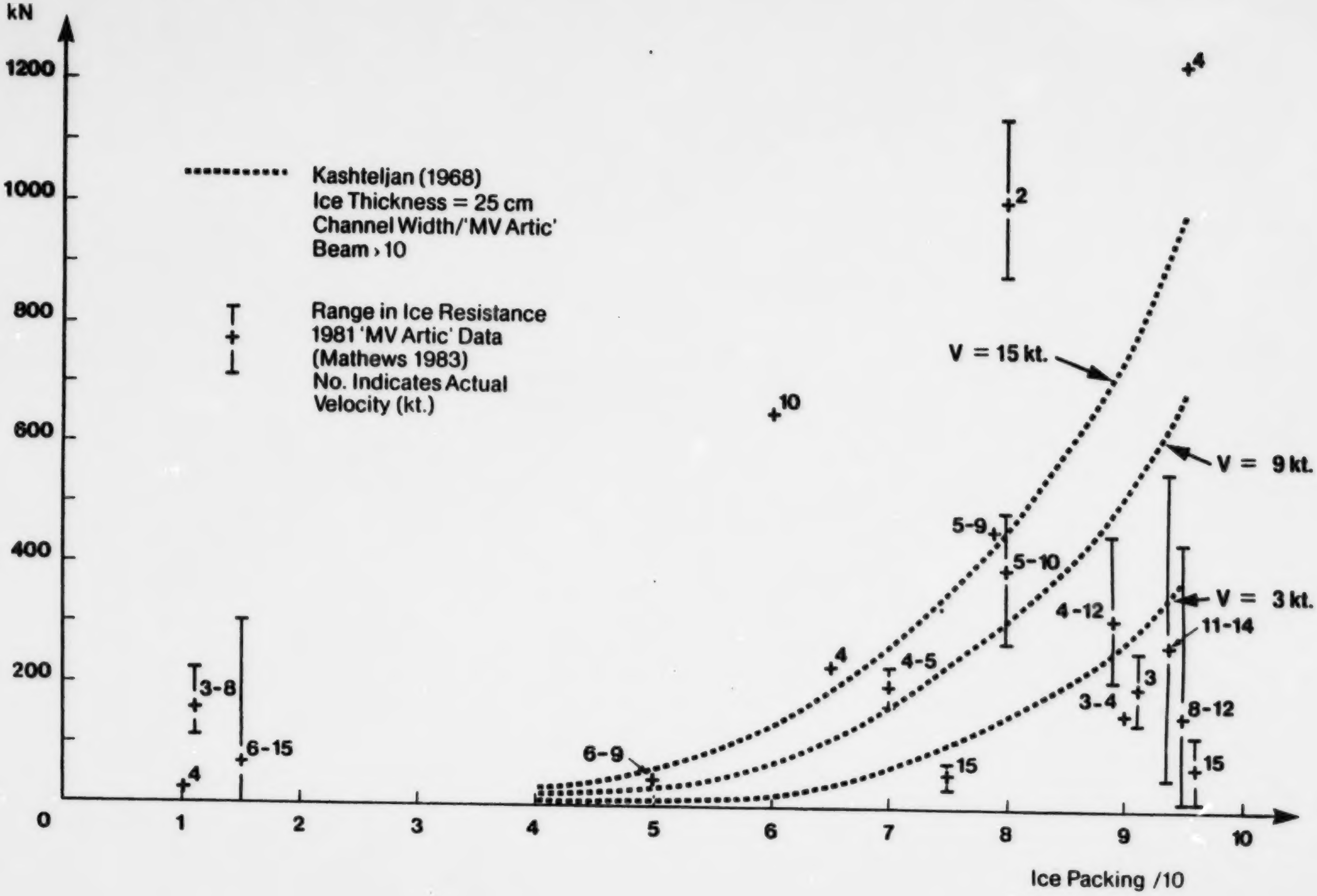


Figure 5.19 'MV Arctic' Broken Ice Resistance Comparison with Empirical Predictions

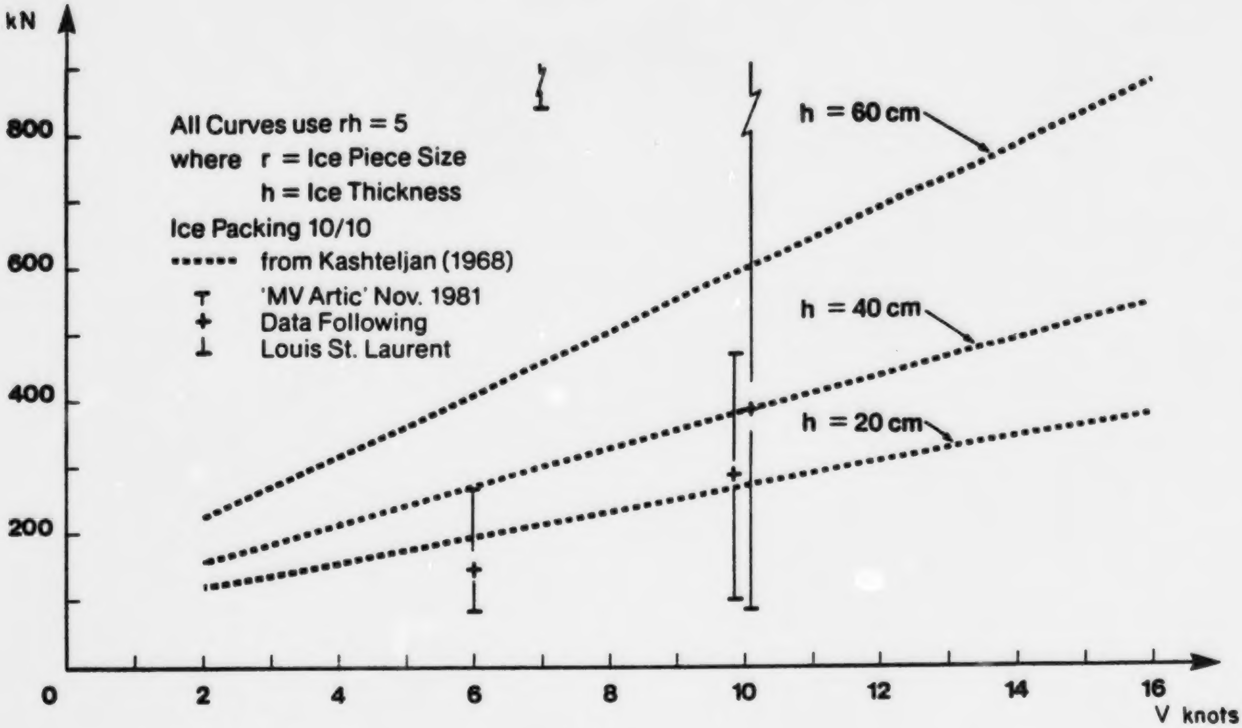


Figure 5.20 Variation in Predicted 'MV Arctic' Broken Ice Resistance with Ice Thickness

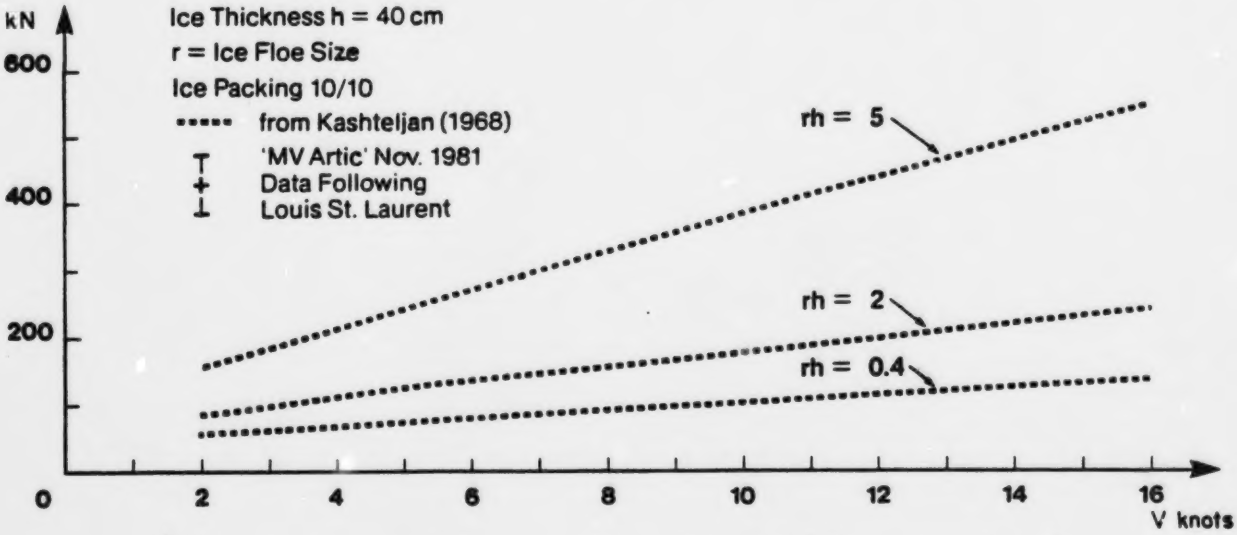


Figure 5.21 Variation in Predicted 'MV Arctic' Broken Ice Resistance with Ice Floe Size

6.0 GRAPHICAL PRESENTATION OF PERFORMANCE

As demonstrated in Section 5, the modified Poznyak and Ionov equation (Eqn. 10) can be accepted as being a reasonable formulation of the performance of the M.V. Arctic in continuous level sheet ice during the various trials carried out in 1979-80. The equation can be used for predicting the performance of the M.V. Arctic with its present bow shape provided the strength of the ice to be traversed and the hull/ice dynamic friction coefficient can be assessed with reasonable accuracy.

Since Equation 10 in its full form takes the bow angles into account, it should also be applicable to changes in bow shape unless such changes are of a radical unconventional form. The equation should, however, be tested against the model test results for an altered bow shape. It is also emphasized that Equation 10 is only one of several types of analytical equations, many of which can be made to fit limited data by modification of parameters, coefficients and indices. The structure of the "ideal" equation for general prediction has yet to be demonstrated by further fundamental analysis and testing against the widest possible range of full-scale trials and model experiments.

The performance of the M.V. Arctic as given by Equation 10 is shown in Figures 6.1, 6.2 and 6.3 in the form of the relationship betwen total resistance, speed, power and ice thickness for a flexural ice strength of 350 kPa and three different coefficients of friction between hull and ice.

For other ice strengths, the power or thrust requirement can be obtained by raising or lowering the ice thickness curve according to the following formulae based on the first term of Equation 10.

$$R_b\ (\sigma_f) - R_b(350) = C_b h^2\ (\sigma_f - 350)$$

or

$$\Delta R_b = C_b\ h^2 \Delta\ \sigma_f$$

Where R_b (σ_f) is the breaking resistance for the actual strength of the ice. The coefficient C_b varies with friction so that when:

$f_d = 0.03$, $C_b = 3.31$
$f_d = 0.08$, $C_b = 4.55$
$f_d = 0.13$, Cb = 5.57

Figures 6.1 to 6.3 show performance for different power levels and each figure assumes a different hull/ice friction coefficient. Figure 6.4 presents, in a single diagram, speed versus ice thickness under full power and a procedure for adjusting for changes in ice strength and interpolating the effect of friction.

A dynamic friction coefficient of 0.03 fits well with the dedicated trials data and is a reasonable value for a hull coated with Inerta 160 or equivalent and for ice with negligible snow cover. At a reasonable speed of 2 m/s (3.9 knots) for continuous progress through ice, Figure 6.1 indicates the following relationship between ice thickness and required power for a vessel of the size and shape of the M.V. Arctic.

Ice thickness 0.61 m required power 5,600 kW
Ice thickness 0.85 m required power 11,000 kW
Ice thickness 1.00 m required power 16,500 kW

The data base is still inadequate to produce a reliable formula for the effect of snow, although Carter[41] proposes the addition of a term 900 h_s in kilo Newtons where h_s is the depth of snow in metres. Carter also suggests the possibility of adding to the friction coefficient but, before this could be done with confidence, much more would need to be known about the proportional effect of snow cover on the velocity and non-velocity components of resistance. Pending further knowledge, a simple addition of snow resistance to the computed ice resistance would seem to be the best procedure.

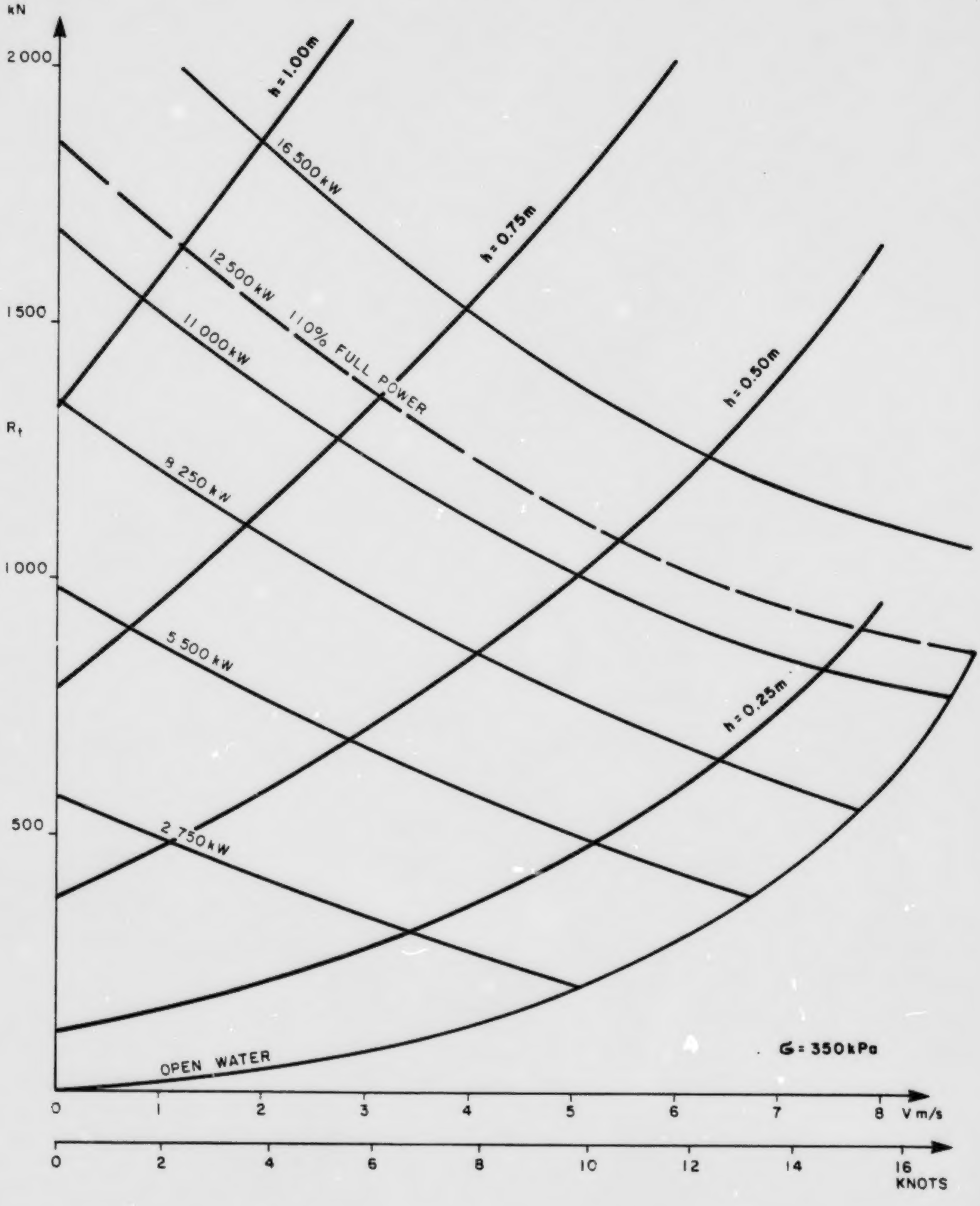


FIGURE 6.1 M.V. ARCTIC SHEET ICE PERFORMANCE DIAGRAM FOR f_d = 0.03

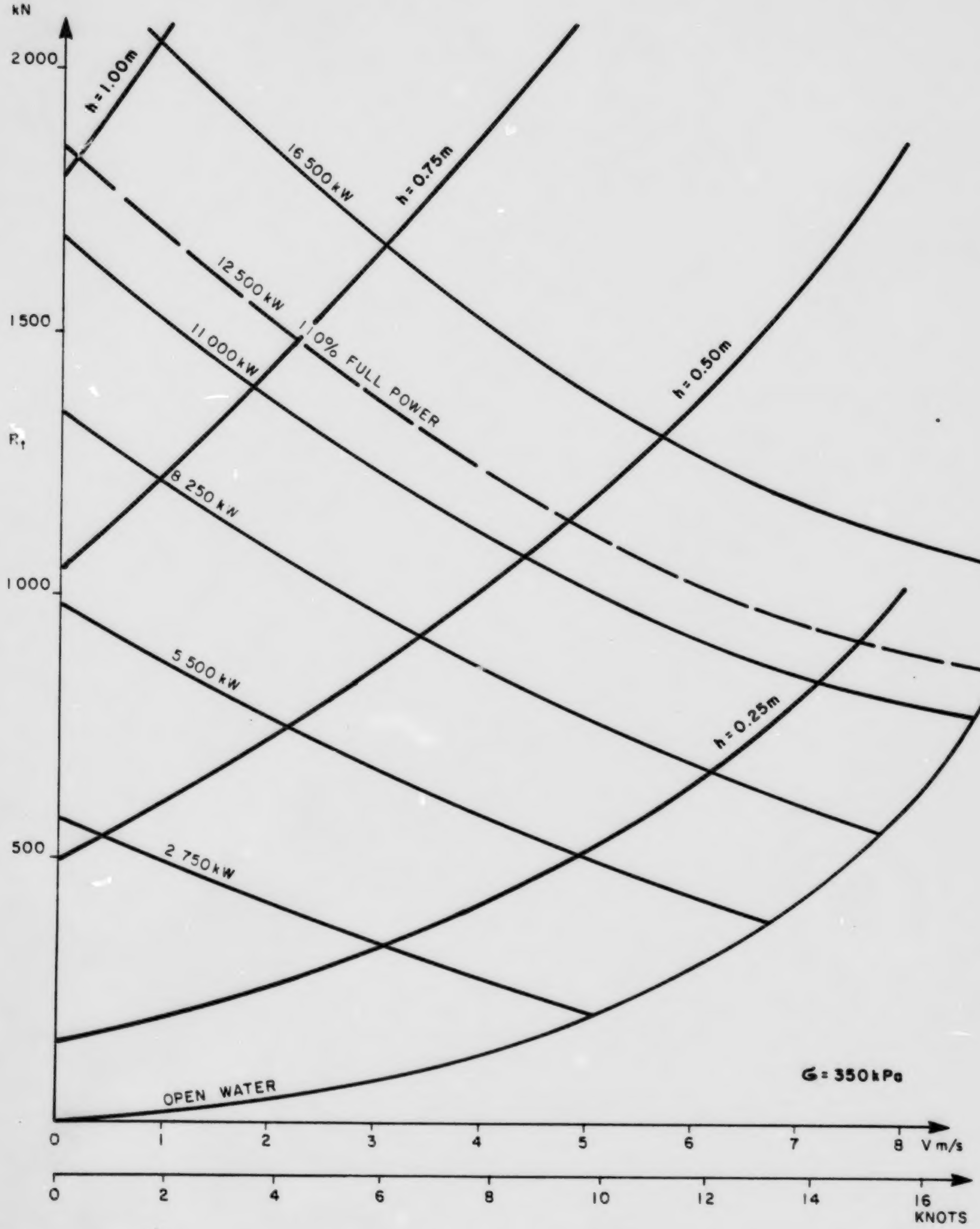


FIGURE 6.2 M.V. ARCTIC SHEET ICE PERFORMANCE DIAGRAM FOR f_d = 0.08

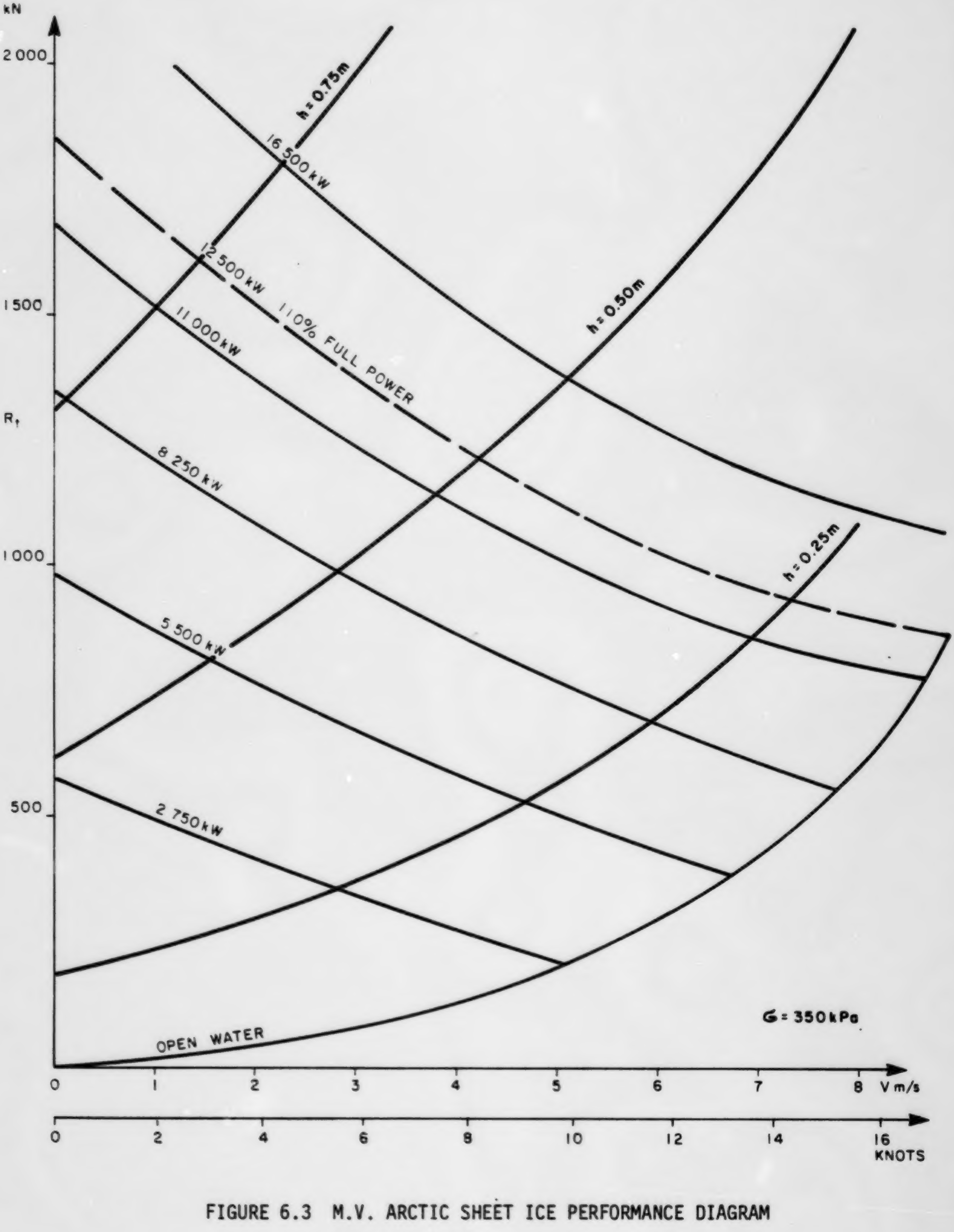


FIGURE 6.3 M.V. ARCTIC SHEET ICE PERFORMANCE DIAGRAM FOR $f_d = 0.13$

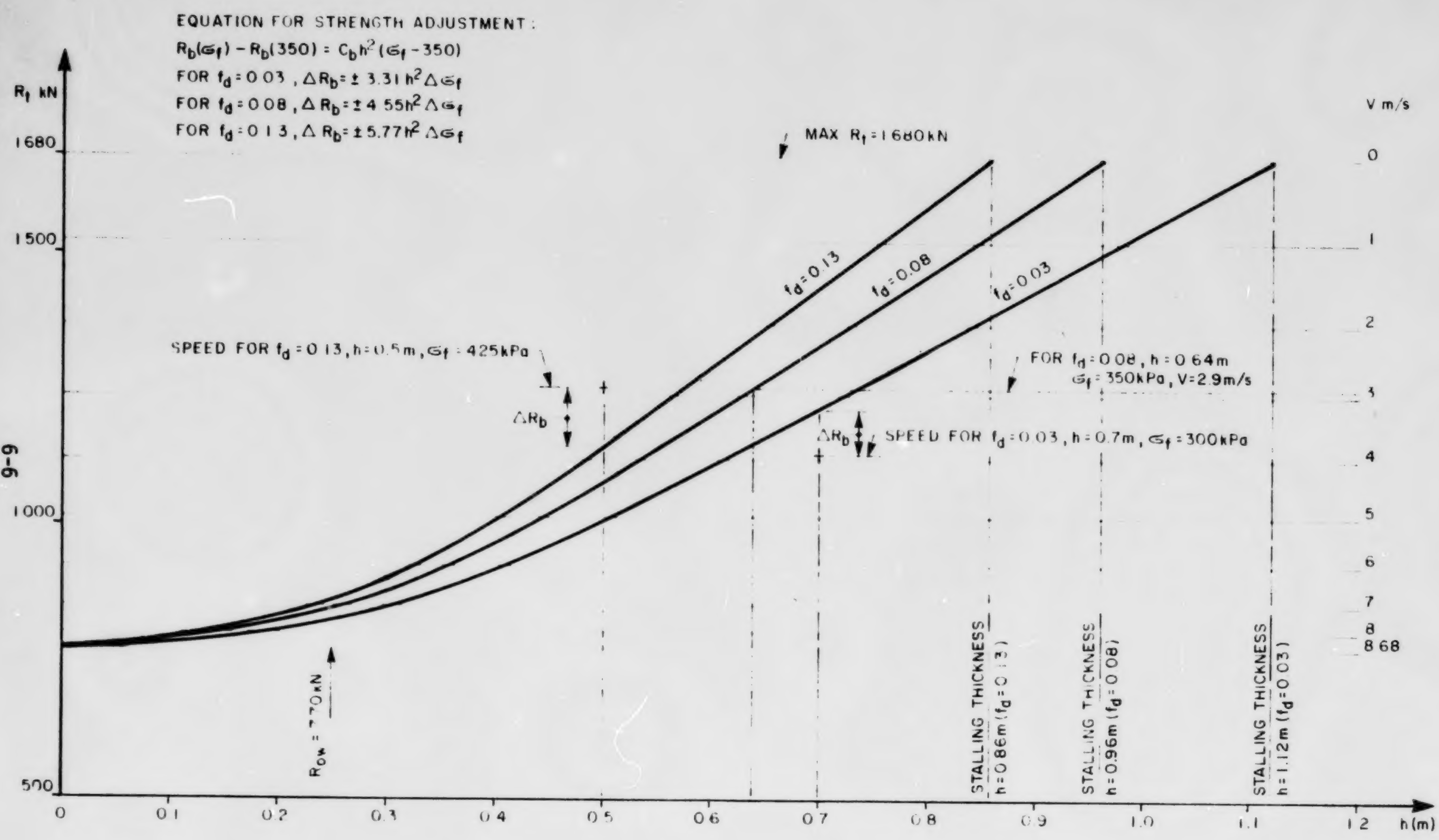


FIGURE 6.4 M.V. ARCTIC FULL POWER SPEED VERSUS ICE THICKNESS DIAGRAM

7.0 POWER PROVISIONS OF CASPPR

The M.V. Arctic is an Arctic Class 2 vessel. The installed power of 11014 kW is 17% more than that required as a minimum for a Class 3 ship according to Schedule G of the Regulations and 2.6 times that required as a minimum for a Class 2 ship. The astern power level requirement is 70% of that ahead and this requirement is met with similar factors to those ahead.

The performance of the vessel in level ice is now well defined from the analysis of the Lake Melville, Baffin Island and Saguenay River trials. This is shown in Figures 6.1 to 6.3 for an assumed ice strength of 350 kPa and a range of hull/ice friction coefficients. For level sheet ice with negligible snow cover, negligible internal pressure in the ice field and with a hull coating of Inerta 160 in good condition the performance is represented by Figure 6.1. This indicates a maximum continuous capability to transit 1.0 m ice at 1.6 knots. However, 1.6 knots is too slow for continuous operation through ice and a more realistic ice thickness - power relationship is that given at the end of Section 6.0 for a speed of 3.9 knots.

In addition to the power requirements for continuous motion in level sheet ice, it is relevant to consider the definition of Arctic Classes according to the limiting seasons for operating into permitted zones.

The M.V. Arctic's limiting operation as a Class 2 ship is into Zone 13, entering no earlier than June 25 and leaving no later than November 15 in each year. If she were classified as an Arctic Class 3 ship, the earliest entry into and departure from Zone 13 would be June 10 and December 31, respectively.

In the approach to and departure from Nanisivik during Voyage 1, powers close to full installed power were used for any continuous progress at very slow speeds. Similar results were obtained for Voyage 5, even in the case of the M.V. Arctic following the CCGS Louis St. Laurent.

In Voyage 5 propeller thrusts often exceeded 800 KN corresponding to shaft powers exceeding 9500 kW. These thrust levels were associated with as many as 20 thrust level changes in 15 minutes associated with the necessity for backing and ramming and this following the CCGS Louis St. Laurent.

For this Class 2 operation it is clear that the CASPPR minimum requirement of 4174 kW of installed power is inadequate.

It is also clear that a Class 3 operation with extended dates earlier than Voyage 1 and later than Voyage 5 with a CASPPR power of 9390 kW could not have been carried out unassisted.

It is evident, therefore, that the minimum CASPPR power requirements for Classes 2 and 3 are too low by a large factor for a cargo vessel of the shape of the M.V. Arctic operating within the seasons now permitted for Zone 13.

It may be concluded, therefore, in any re-drafting of the Regulations using the concept of an equivalent level ice thickness in which each Class must operate at some designated speed, the nominal ice thickness for Classes 2 and 3 should be increased to better represent the ice conditions in Zone 13 during the permitted seasons.

In addition, for consideration of revised regulations, the prediction of level ice performance of the M.V. Arctic should include some means for extending to other ship and ice scales and some changes of form.

The installed power of the M.V. Arctic would seem to be adequate for the ice conditions, thicknesses, strength and elasticity encountered in the Lake Melville trials, in order to maintain a sufficient margin of speed above the creep speed.

The actual installed power would seem to be barely adequate for the first of the early season and last of the late season for the Zone 13 Class 2 operation, even with a very effective propulsion nozzle arrangement. It is to be hoped that this situation will be improved with the re-design of bow.

It would seem reasonable, with Regulations giving rise to adequate powers for the various Arctic Classes of ships (operations - Zones and Seasons), to give credit for demonstrably good propulsive-nozzle arrangements.

The effect of realistic limiting values of ice elasticity, strength and friction should also be considered in formulating new criteria.

As a point of detail, the existing CASPPR Regulations consider a reference propeller diameter, Dr. The formula for this does not give the units. This is ambiguous and misleading. Dr, as given, is in feet and this should be stated.

The economy of operation is partly dependent on the ratio resistance/displacement both in ice and open water for various speeds. This is proportional to the energy expended per unit of displacement per mile, i.e. fuel utilization. This is always lower at lower speeds. However, at lower speeds the rate of earning is lower (fewer voyages per season). These considerations, together with many others - capital cost, depreciation, maintenance, wages, insurance lead to a full economic analysis of the effect of speed and power. This is beyond the scope of the present work.

8.0 MODEL VERSUS FULL-SCALE DATA

8.1 Introduction

Some of the general problems related to the realistic modelling of icebreaking resistance have been discussed in Section 2. Whilst it is claimed that the particular problem of simultaneously modelling the correct values for strength and strain modulus has been overcome by the recent use of carbamide ice, there remains considerable doubt regarding the quantitive accuracy of model tests. These doubts have been reiterated as recently as May 1983 by Keinonen[42] in a paper prepared for the 20th ATTC. In it he compares full-scale results from the icebreaker Kigoriak with model tests made about four years ago by Arctec and Wartsila. He concludes that:

1. Model tests tend to over-estimate the extent of propeller-ice interaction and the risk the nozzle (duct) being blocked by ice.

2. There were serious weaknesses in the state of the art of ice resistance modelling as it existed four years ago. In spite of the introduction of carbamide ice and other improved ice material any attempt to optimize designs using model tests should include skillful analytical interpretation as well as routine scaling.

The comparison of the results of the M.V. Arctic trials with the 1974 model tests carried out by Arctec should be viewed in the light of the above comments.

Table 8.1 reproduces the essential data from the 1974 Plate Ice Resistance Model Tests carried out by Arctec Canada Ltd. Columns 1 to 6, 8 and 9 are conversions to full-scale values in S.I. units from the original Arctec data. Column 7 has been added.

TABLE 8.1

DATA FROM 1974 MODEL TESTS

1 Sheet Test	2 R_t, kN	3 h, m.	4 σ_f kPa	5 v, m/s.	6 $\frac{R_t}{\gamma_\omega B h^2}$	7 $\frac{R_i}{\gamma_i B h^2}$	8 $\frac{V}{\sqrt{gh}}$	9 f_d
2-1	5,740	1.00	380	4.20	25.3	26.44	1.34	0.272
3-1	2,133	0.52	445	4.18	34.9	31.72	1.83	0.272
3-2	1,373	0.48	445	1.01	27.0	28.48	0.48	0.272
4-1	11,026	1.43	1503	2.01	24.1	25.37	0.54	0.272
4-2	11,387	1.45	1503	2.06	23.9	25.82	0.54	0.272
4-3	10,747	1.47	1503	0.93	22.1	23.46	0.24	0.272
4-4	9,470	1.32	1503	0.95	24.3	25.24	0.26	0.272
5-1	2,403	0.61	858	3.15	29.4	29.65	1.30	0.272
6-1	2,627	0.59	825	3.14	33.0	34.26	1.30	0.132
6-2	1,675	0.58	825	0.97	22.5	23.31	0.41	0.132
7-1	2,119	0.45	701	5.69	46.8	42.94	2.71	0.132
7-2	1,158	0.40	701	2.06	33.0	32.98	1.05	0.132
8-1	1,282	0.66	1,055	3.14	13.0	12.71	1.23	0.03
8-2	677	0.62	1,055	0.99	7.88	8.06	0.40	0.03
8-3	398	0.64	-	3.14	4.33	3.64	1.25	0.03

NOTE: Test 4-1, Yaw restraint broke. Not included in regression.
Test 4-3, Short Run. Not included in regression.
Test 8-3, Broken Channel. Not included in regression.

Source: "Plate Ice Resistance Model Tests on a 28,000 DWT Arctic Class 2 Bulk Carrier" by Arctec Canada Ltd. 1974.

Figure 8.1 shows the dimensionless ice resistance, $R_i/\gamma_i Bh^2$ plotted against the dimensionless velocity and compared with the full-scale test data. This figure shows that:

(a) The two valid tests, 8-1 and 8-2, with an assumed hull/ice friction coefficient of 0.03 lie on the general line of the full-scale data. However, the full-scale strength of the model ice was 1036 kPa, about three times that of the actual ice in the full-scale trials and the points should have plotted much higher. This suggests that, either the model friction coefficient was substantially less than the prototype coefficient or the relationship between model ice strength and resistance is not the same as that between real ice and resistance. This is discussed later in subsection 8-2.

(b) The six valid tests 2-1 to 5-1 lie roughly on the same line in spite of the fact that the full-scale strength of the model ice varied from 373 kPa to 1475 kPa. This suggests that there are properties in the model ice which, either do not give the correct model strength with the measuring procedures used, or do not give a breaking resistance varying linearly with σ_f as generally assumed. This is discussed later in subsection 8-3.

(c) The four tests 6-1 to 7-2 lie very roughly on the same general line as tests 2-1 to 5-1 in spite of the fact that the former have a stated friction coefficient of 0.132 compared with 0.272 for the latter. This is discussed below.

8.2 Discussion on Friction

Figure 8.2 reproduces Figure A.1 from Appendix A of the Arctec report which was used to derive the apparent values of f_d

from the friction test data. It can be seen that the lines drawn through the origin do not fit well with the test points plotted. The dashed lines superimposed by APD give a better fit, and, if the origin of the normal force is correct, the tests indicate that:

$$R_t = C + Nf_d$$

where R_t is the towing force or total resistance
N is the normal force causing friction
and C is a constant for a particular surface

There is, however, some reason to believe that there may be an error in the normal force and the location of the origin in Figure 8.2. Page A.3 of Arctec's Appendix A states that "the normal load was varied from near gravity force to approximately 1.5 times gravity", whereas Figure A.1 shows a variation of just over 2.0 times. If the dashed lines are taken back to a new origin about 100 grams behind the origin shown, the 1.5 times variation would apply. This would mean that a weight of 100 grams has been omitted inadvertently from the total normal force used in the tests.

Another question is the adequacy of the range of 110 to 230 grams (or 210 to 330) used in the friction test. The model resistance is as much as 24,880 grams of which 35% or 8,700 grams might be due to friction (See Pozniak, Ref. 24) requiring a total normal force of about 58,000 grams for $f_d = 0.15$. Arctec (Pages 8 and 9) state that there is some crushing of ice at the cheeks of the bow and that "the cusps formed by the bow are crushed to form flattened points along the sides of the ship". This would suggest large normal ice forces in local areas far beyond the 1.5 times gravity tested.

The large local ice forces during crushing might explain why tests 6-1 to 7-2 appeared to indicate the same hull/ice friction as tests 2-1 to 5-1 in spite of the difference measured in the friction tests. The measurements were made on the bottom of the vessel. The surface for tests 2-1 to 5-1 was created by wood filler, two coats of epoxy paint, two coats of polyurethane varnish and wet sanding perpendicular to ship motion with 600 grit sandpaper. The surface for tests 6-1 to 7-2 was as above but waxed with automotive paste wax and buffed to a high gloss. The effect of the wax would probably be unaffected on the bottom under a test weight of only 1.5 times gravity. But its effect might be rapidly destroyed on the sides of the vessel under the local abrasive forces caused by crushing of the ice.

A further point to bear in mind when assessing the effective friction of the model is that, during breaking, any ice failing by bending moves down the sides of the bow so that friction acts in a vertical direction as well as in the direction of motion of the ship. Since the sanding was done perpendicular to motion and the tests were made only parallel to motion, the overall effective coefficient may not have been truly measured and may be somewhat less than the value indicated by Figure 8.2.

To summarize the question of friction three points emerge from the foregoing discussion.

(1) The true values of f_d given by the friction tests may be about 0.169 and 0.071 rather than 0.272 and 0.13 given by Arctec.

(2) The wax coating applied to reduce the friction may have been ineffective, so that all tests except 8-1 and 8-2 may have had a friction coefficient of about 0.169.

(3) The sanding and testing in directions at right angles to each other may not give a true effective friction coefficient since friction acts vertically as well as horizontally. (See Figure 9.1)

8.3 Correlation of Equation 10 with Model Tests

As described in Section 5, Equation 10 is based on the form of the Pozniak and Ionov equation [24] and makes use of their friction/form relationships. The numerical coefficients and the index of h in the third term have been altered, however, to correlate with the full-scale trials data of the M.V. Arctic. In what follows in this Section Equation 10 is examined to test its ability to correlate with the model test data.

In its simplified form Equation 10 is:

$$R_t = 0.035\ \sigma_f h^2\ \{\mu_1\} + 0.029\ \gamma_\Delta\ hBL\ \{\mu_2\} + 7.4\gamma_i B\left(\frac{H}{L_w}\right)^{\frac{1}{2}} h^2 . \frac{V}{\sqrt{gh}}\ \{\mu_3\} + R_{o\omega}$$

where μ_1, μ_2 and μ_3 are friction/form factors. As pointed out previously, μ_1 is dimensional and consists of $a_1(\beta)B + f_d a_2(\beta)L$ where $a_1(\beta)$ and $a_2(\beta)$ are non dimensional form coefficients as defined in Appendix A, page A-32. For proper comparison of ships of different sizes and when using model dimensions the first term should be written in a form such as

$$1.17\sigma_f h^2\ \frac{a_1\ (\beta)\ B + f_d\ a_2\ (\beta)\ L}{L_w}$$

where L_w is the cube root of the volumetric displacement = 33.36 m. and the new numerical coefficient, 1.17 = 0.035 x 33.36. However, if all model dimensions are scaled up to the M.V. Arctic prototype dimensions in S.I. units, Equation 10 can be used without change.

Figure 8.3 is a reproduction of Arctec Figure 2.1.1 which plots dimensionless resistance $\frac{R_t}{\gamma_\omega B h^2}$ against $\frac{V}{\sqrt{gh}}$ from columns 6 and 8 of Table 8.1. Superimposed on this Figure is a dimensionless plot of Equation 10 using the following values for the variable parameters.

σ = 690 kPa (100 psi as used by Arctec for their predictor)
h = 0.86 (average for tests 2-1 to 7-2)
f_d = 0.169 (as derived in subsection 8-2)
$\gamma\omega$ = 10.06

It can be seen that, with the above assumptions, Equation 10 fits remarkably well with the model data from tests 2-1 to 7-2, - much better, in fact, than the predictor shown by Arctec.

Also superimposed on Figure 8.3 is a plot of Equation 10 for comparison with tests 8-1 and 8-2 using the following values:

σ_f = 1036 kPa (4.17 x 36 psi)
h = 0.64 m. (1.78 cm x 36)
f_d = 0.03

As can be seen, the plot of Equation 10 lies above the two experimental points which suggests either one or a combination of the following explanations:

(1) That the hull/ice dynamic friction coefficient for the very smooth polyurethane varnish finish was less than the 0.03 assumed.

(2) That the effective model strength is much less than measured or that the resistance is affected by other parameters, such as the strain modulus, E, of the ice and the size of broken ice pieces as well as the strength. Values of E were measured but are not given in the Arctec report and have not been obtainable to-date. Size of ice pieces in relation to ice thickness does not appear to have been recorded.

The significance of item (2) above in relation to the way in which the values of σ_f and E are obtained in both model and full-scale tests requires further study beyond the scope of this report.

The predictor equation derived by Arctec from regression analysis of all data except tests 4-1, 4-3 and 8-3, is in dimensionless form, -

$$\frac{R_t}{\gamma_\omega B h^2} = 0.523 \frac{\sigma_f f_d}{\gamma_\omega h} + 4.90 + 10.42 \frac{V}{\sqrt{gh}}$$

Figure 8-3 appears to indicate that the Arctec equation predicts too low values of dimensionless resistance at low Froude numbers. This is because of the omission of Tests 4-1 and 4-2 from the regression analysis and the effect of the very low values for Tests 8-1 and 8-2. It also reflects the questionable fact that the model tests were done with three stated hull/ice friction coefficients whereas a single predictor equation is given a coefficient of 0.15.

Table 8.2 shows a direct comparison of the model test resistance with that given by Equation 10 using the same values of V, h and σ_f but the new derived value of f_d = 0.169 for tests 2-1 to 7-2. Whilst, as shown in Figure 8.3, Equation 10 correlates well with the average of the model test results, there is a wide divergence in some cases, particularly in tests 4.1 to 4.4 where the model ice had both an above average thickness and a high strength. The model resistance in tests 4.1 to 4.4 of only half that given by Equation 10 may be due to the special properties of saline model ice but it may also cast doubt on the range of applicability of an equation having σ_f and h^2 in the same term. This reinforces the need for testing Equation 10 against other full-scale and model tests over the largest possible range of ice thickness.

TABLE 8.2

MODEL TEST RESISTANCE VALUES COMPARED WITH EQN. 10

Sheet Tests	Value of R_t in kN	
	Model Tests by Arctec	Calculations from Eqn. 10
2-1	5,740	4,143
3-1	2,133	1,589
3-2	1,373	905
4-1	11,026	21,700
4-2	11,387	22,595
4-3	10,747	22,453
4-4	9,470	18,053
5-1	2,403	2,799
6-1	2,627	2,622
6-2	1,675	2,102
7-1	2,119	1,872
7-2	1,158	1,033
8-1	1,282	1,994
8-2	677	1,661
8-3	398	--

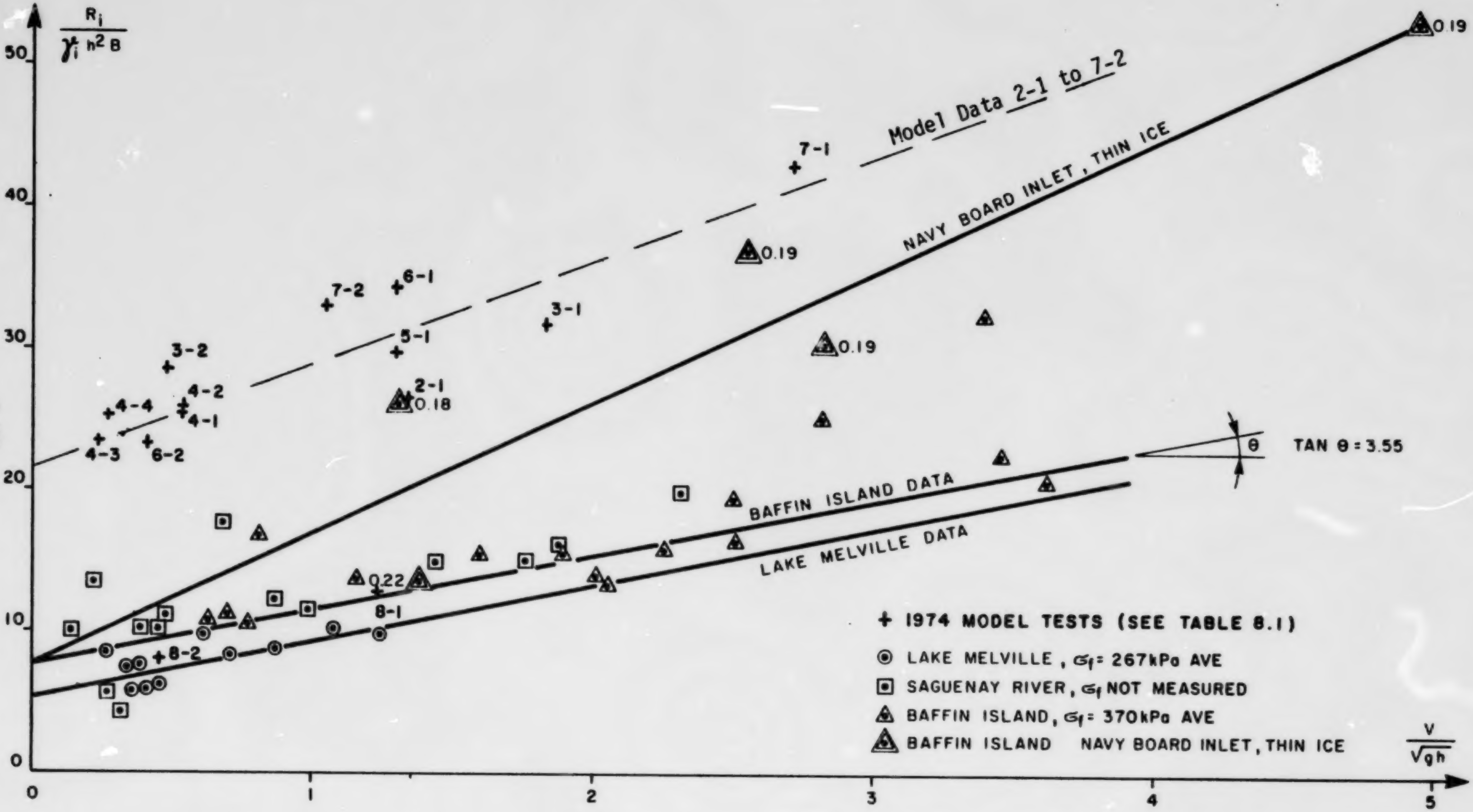


FIGURE 8-1. DIMENSIONLESS ICE RESISTANCE VS. DIMENSIONLESS VELOCITY COMPARING MODEL TESTS WITH FULL-SCALE DATA.

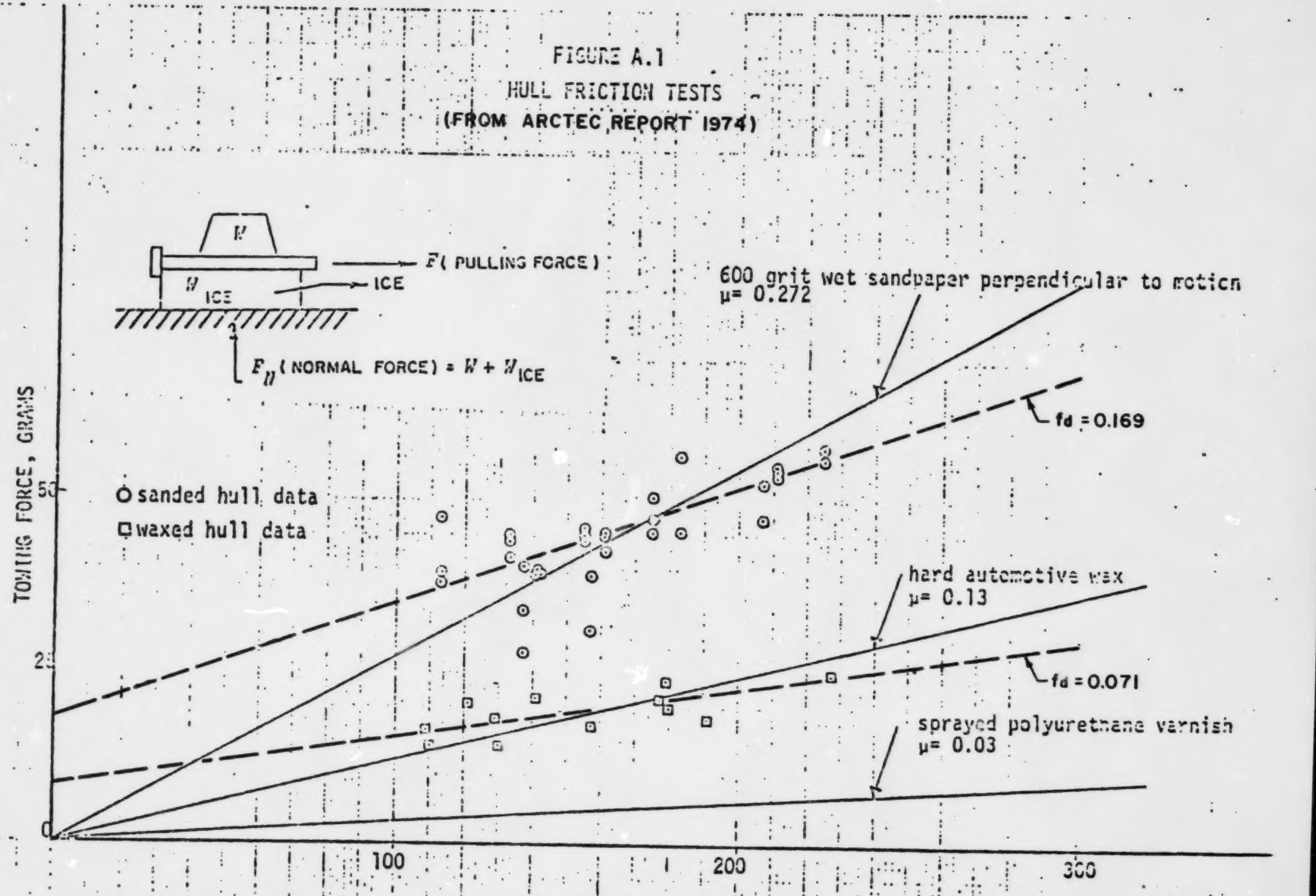


FIGURE 8.2 MODEL FRICTION TESTS – ALTERNATIVE INTERPRETATION

FIGURE 2.1.1

DATA PLOT, DIMENSIONLESS RESISTANCE VERSUS DIMENSIONLESS SPEED

(FROM ARCTEC REPORT 1974)

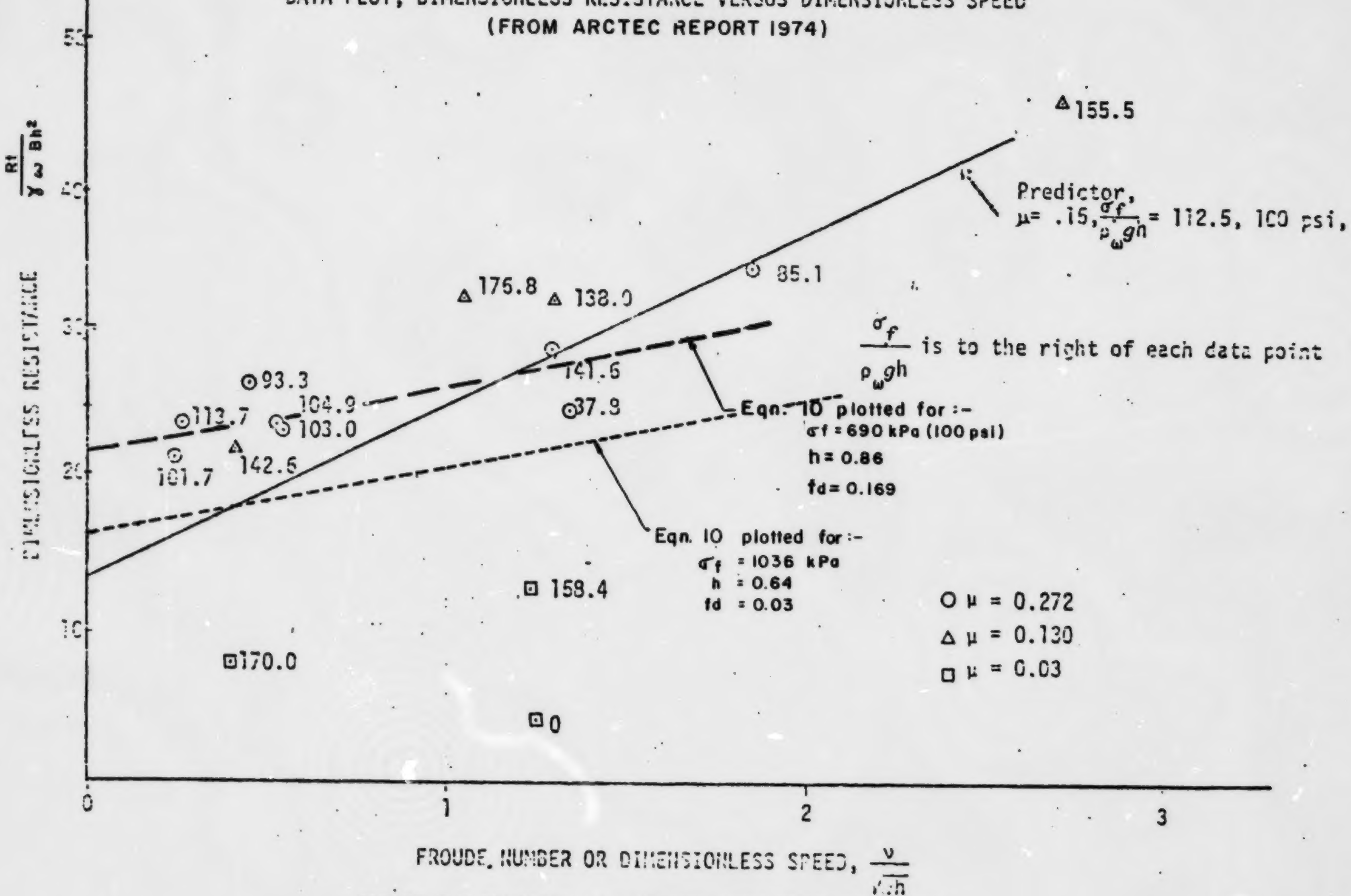


FIGURE 8.3 EQUATION 10 COMPARED WITH MODEL TEST DATA

9.0 HULL GEOMETRY

During the past several years, there has been a growing recognition of the importance of hull geometry and friction in relation to the ice-breaking process. Concurrently, it has become apparent from model and full-scale tests 24 & 28 and new analytical approaches that the breaking component of total resistance is much greater than originally supposed by researchers such as Kashtelyan and Enkvist.

Until recently most emphasis was placed on the shape and angle of the ice-cutting stem. However, in the case of a large vessel, the ice-cutting stem does not usually break the ice over the whole width of the bow and the sides of the bow have to widen the channel by breaking off cusps of ice as the vessel proceeds. The icebreaking resistance may be said therefore, to consist of two parts:

1. $R_{st.}$ the initial breaking of the ice by the stem.
2. $R_{sh.}$ the subsequent breaking off of ice cusps by the shoulders

Referring to Figure 9.1 the forces involved in the shoulder resistance, R_{sh}, are as follows:

Breaking Moment = resistance moment of ice

$$P\ell\sin\delta - fP\ell\cos\delta = Ch^2\sigma$$

$$\therefore \quad P = \frac{C\,h^2\sigma}{\ell\,(\sin\delta - f\cos\delta)}$$

WHERE: P = force normal to bow bending ice.

f = coefficient of friction.

ℓ = effective lever arm of breaking force.

δ = angle of ship side to vertical at ice level.

C = a coefficient depending on shape of ice cusps

h = ice thickness.

σ = ice flexural strength.

In the case of a simple cantilever beam the constant C would be 0.167 per unit width. Enkvist suggests that for a cusp forming part of a semi-infinite plate on an elastic foundation, C would be about 0.67 per unit length.

The static or instantaneous resistance to motion of ship per unit length of ice contact is 2R where:

$$\begin{aligned} R &= P \sin\alpha + f P \cos\alpha \\ &= P\ (\sin\alpha + f \cos\alpha) \\ &= \frac{Ch^2\sigma}{\ell} \frac{\sin\alpha + f \cos\alpha}{\sin\delta - f \cos\delta} \end{aligned}$$

In theory, R is infinite and no bending force is possible if:

$$f \cos\delta > \sin\delta$$

or

$$f > \tan\delta$$

In practice, if f is too great or δ too small, the ice will break in shear or buckling rather than in bending.

The total static, or instantaneous shoulder breaking resistance is:

$$2\, \Sigma_a^b R = \frac{2\, C'\, h^2\, \sigma}{\ell}\ \Sigma_a^b\ \Delta L_{sh}\ \frac{\sin\alpha + f \cos\alpha}{\sin\delta - f \cos\delta}$$

where ΔL_{sh} is the incremental length of shoulder to which the angles are applied and C' is dependent on shape of ice cusps.

The total dynamic breaking resistance on the basis of energy per unit length is:

$$R_{sh} = 2\, C''\, h\sigma \frac{\sigma}{E}\ \Sigma_a^b \left\{ \Delta L_{sh}\ \frac{\sin\alpha + f_d \cos\alpha}{\sin\alpha - f_d \cos\delta} \right\}$$

The obvious conclusion to be drawn from the above equation is that the shoulder resistance to breaking may be considerable and only limited by shear or buckling strength of the ice if the friction is too great in relation to the vertical angle of the shoulders.

Relating the above illustrative but incomplete analysis to the M.V. Arctic, it can be seen at once that the small to zero flare at the shoulders will not facilitate breaking in bending over this area of the bow and will tend to cause jamming in the ice and increased resisting forces. Those forces at the shoulders of the vessel may not be very significant in thin ice where the ice can easily buckle but may become increasingly great as the vessel proceeds into thicker ice where considerable crushing may result. The low friction coating of Inerta 160 will reduce the resistance to motion and may be particularly effective in ice that is too thick to buckle readily.

The effect of hull geometry and friction has recently been analyzed by Carter[41] and his analysis has been applied to a number of ice-breaking vessels, including the M.V. Arctic. From his mathematical model the following general trends are indicated:

Forebody Characteristics	Influence on Level Ice Resistance
Ship Beam, B	Intermediate
Ratio Forebody Length/Beam	Intermediate
Stem Angle, ϕ	Intermediate for $\phi < 30^\circ$
	Strong for $\phi > 30^\circ$
Spread angle, α_o	Small
Flare angle at midship, β	Strong for $\beta < 15^\circ$
	Intermediate for $\beta > 20^\circ$
Coefficient of friction, f	Strong

Carter gives the following angles for the optimum hull compared with those of the M.V. Arctic:

Angle	Optimum	M.V. Arctic
Stern	18°	30°
Bow entrance (Spread)	$2 \times 45^\circ$	$2 \times 32.8^\circ$
Midship flare	20°	0°

The bow entrance angle is of minor importance and the stern angle of 30° for the M.V. Arctic is a reasonable compromise for a cargo vessel designed to pass through the St. Lawrence Seaway locks. Passage through the locks also puts a constraint on the mid-body flare angle but Carter's analysis confirms the importance of this angle in reducing ice resistance. Carter calculates that for $h = 1$ m, $\sigma = 800$ kPa and $f = 0.2$ the ice resistance of a vessel with "optimum hull" and zero flare at midship might be about 4.6 times the ice resistance of a vessel with "optimum hull" and 20° flare. This ratio seems incredibly high but would probably be much less if a value of $f_d = 0.03$ is used to allow for an Inerta 160 coating.

The fundamental requirement for any low resistance icebreaking bow is to break as much of the ice as possible in pure bending to the full width of the vessel and, in this regard, the minimum flare angle is critical for a conventional type of ice-breaking bow. Improvements to the bow of the M.V. Arctic may, however, be constrained by its function as an ocean-going bulk carrier and by its need to pass through the locks of the St. Lawrence Seaway.

Model tests have recently been carried out to develop an improved bow shape for the M.V. Arctic and it would therefore be presumptive to suggest specific improvements in this report. It would be worthwhile, however, to make the test data available in due course so that the recommended predictor equation may be checked against the new bow shape and an improved equation developed as may be desirable.

The shape of the hull astern of its section of maximum water-line beam is relatively unimportant as far as ice transiting under normal conditions is concerned. However, the long length of the M.V. Arctic may result in greatly increased resistance under wind or current conditions that produce internal pressures in the ice field. Such conditions appear likely to have been present during tests on November 26, 1980 in thin ice in Navy Board Inlet.

There is evidence that the draft of the vessel has some effect on the velocity dependent resistance and this is taken into account in Equation 10.

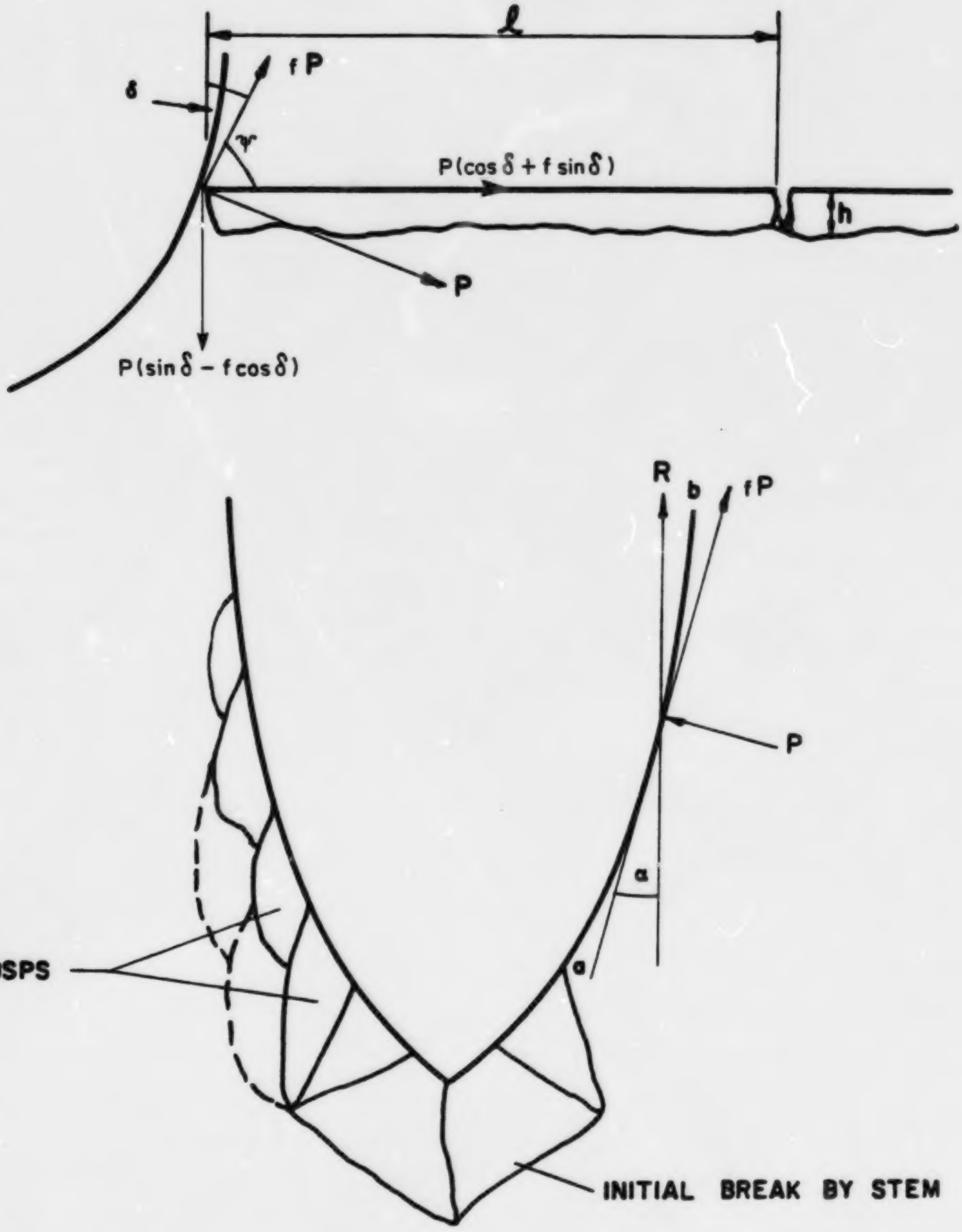


FIGURE 9.1 ICE BREAKING FORCES ON SHOULDERS OF BOW

10.0 EFFECT OF BUBBLER SYSTEM

The bubbler system on the M.V. Arctic consists of 32 nozzles on each side of the ship discharging through the bilge curve at about 1.5 m above the bottom of the ship and extending from 17 metres aft of the forward perpendicular to about 53 metres forward of the aft perpendicular. On each side there are 16 bubbler nozzles 150 mm diameter at 2.75 m centres along the forward portion of the ship and 16 bubbler nozzles 75 mm diameter at 5.5 m centres along the middle portion of the ship. The first nozzles at the bow are just aft of frame 178 where the flare angle is about 20 degrees.

As can be seen from Figure 5.1, the dedicated trials in Lake Melville indicate that the use of the bubbler system did not provide any reduction of resistance under the conditions tested and, in fact, increased resistance in ice with a thickness of about 0.64 m. The Baffin Island trials data shown in Figure 5.2 also indicates that the bubbler system on the M.V. Arctic is relatively ineffective. The Saguenay River trials data shown in Figure 5.3 appears to show an increase in resistance with bubbler on but this may be misleading in view of the slightly greater ice thickness. The reasons for the ineffectiveness of the bubbler system require detailed analysis but the following possible explanation is offered for further consideration.

In the first place, the essential purpose of a bubbler system is to reduce friction and will obviously be more effective in conditions where hull/ice friction is high. With its Inerta 160 coating, the M.V. Arctic has a naturally low surface friction and the scope for improvement with bubblers is small. Also, nearly all the dedicated trials were carried out in conditions of negligible or small snow cover which again would reduce the potential benefit of bubblers. On the one occasion in Navy Board

Inlet on November 26, 1980, when usually high frictional resistance appears to have been present, the bubblers were not tested.

Another factor contributing to the ineffectiveness of the bubblers may be the vertical sides of the vessel astern of the shoulders. If the vessel has a flare angle, the air bubbles and induced water movement will cling to the surface of the vessel and ensure an effective lubricating film. On the other hand, with a vertical surface the bubbles may dissipate laterally, thus reducing the movement of water over the surface of the hull in the region of the hull/ice contact.

Pending more detailed study, particularly in conditions with internal ice presssure, it is difficult to escape the conclusion that, for a vessel of the shape of the M.V. Arctic with a low friction coating, the use of the bubbler system is a waste of energy.

11.0 EVALUATION OF RIGID BODY MOTIONS

The M.V. Arctic was equipped with six accelerometers and two gyros for the purpose of monitoring its motions. Groups of three accelerometers, positioned to measure the heave, sway and surge motions, were placed both forward and aft on the ship. The two gyros were positioned to measure the motions about the pitch and roll axis. Initially no signal conditioning was used, but at the end of the first year when it was noted that the acceleration signals consisted mostly of noise, a 1.2 Hz low pass filter was added to each sensor. These filtered signals were recorded on new channels, in addition to the raw signals.

Considering the fact that the magnitude, direction and point of application of the external forces acting on the ship cannot possibly be determined, little can be done with the recorded signals except plot them for observation. As a starting point, three runs from the Lake Melville dedicated trials were chosen for plotting. These runs were the continuous mode tests 1b, 4 and 12.

The scaling techniques of the plotting program used to output the performance data proved to be unsuitable in this case due to the relatively high frequency nature of the motion signals and the fact that every channel inlcuded a different steady state offset from zero. After some experimentation, changes were made to expand the horizontal time scale so that each plot covered only 6 minutes; and to remove the bias by subtracting a long term average value of the signal. The second change allowed the use of vertical scales closer to the value of the maximum peak to peak fluctuations.

Figures 11.1 and 11.2 show some typical data plots; in this case for a continuous mode test through 1.05 metre thick ice. Each figure contains three plots; the raw unfiltered signal, the filtered signal and the raw signal after being passed through a

0.01 Hz software digital filter. The raw signal clearly consists mostly of noise, while the filtered signals appear quite reasonable. Close examination reveals the fact that the magnitudes of these accelerations are extremely close to the lower limit of the data system's resolution. The A/D converter has a range of 0 to 1023 counts which corresponds to the acceleration range of -24.5 to +24.5 m/s/s. This means that the smallest acceleration that can be resolved is 0.0458/m/s/s; a value which much of the data falls below. To make matters worse, it was found that the pitch and roll signals were not recorded and that the forward surge signal appeared to be dead. This left very few signals which could be compared as a means of validating the data.

One method that was tried involved the plotting of a few time histories of ramming mode tests. If the accelerometers were working correctly, one would expect the sudden accelerations of the ship to be obvious. To facilitate the validation, an integration routine was added to the program in order to compute velocity from the accelerations. This allowed the direct comparison of the surge velocity with the doppler log ship speed.

The ramming tests chosen for this exercise were also from the Lake Melville dedicated trials. The hardware filtered channels yielded the best results - two examples of which are shown in Figures 11.3 and 11.4. As can be seen, the velocity plots are similar in shape to the doppler log speed plots, but they differ greatly in magnitude. This is most likely due to the fact that all the accelerations are below the resolution of the data system. This is illustrated by the maximum deceleration slope drawn in on the doppler log speed plot of Figure 11.3.

The heave and sway signals were also integrated but yielded very suspicious results as shown in Figure 11.5. Attempts to further integrate these plots to produce displacement yielded totally implausible results.

At this point the investigation was terminated. A summary of the major findings goes as follows:

- Most of the accelerations experienced by the ship were smaller than the resolution limit of the data collection system.
- This was found to be true even in the ramming mode, which one would expect to be more severe than the continuous mode.
- The data recording was incomplete since many channels were frequently absent or inactive.
- Although some channels looked reasonable at times, their accuracy could not be verified.
- Attempts to verify certain channels by computing velocities indicated that the data was most likely false.

Based on the above findings the conclusion was drawn that any observations, with respect to the motions of the ship during the continuous icebreaking mode, could not be made with any confidence. There is a remote possibility that if the investigation were pursued, more positive results could be obtained; however it was felt that any further effort would not be worthwhile.

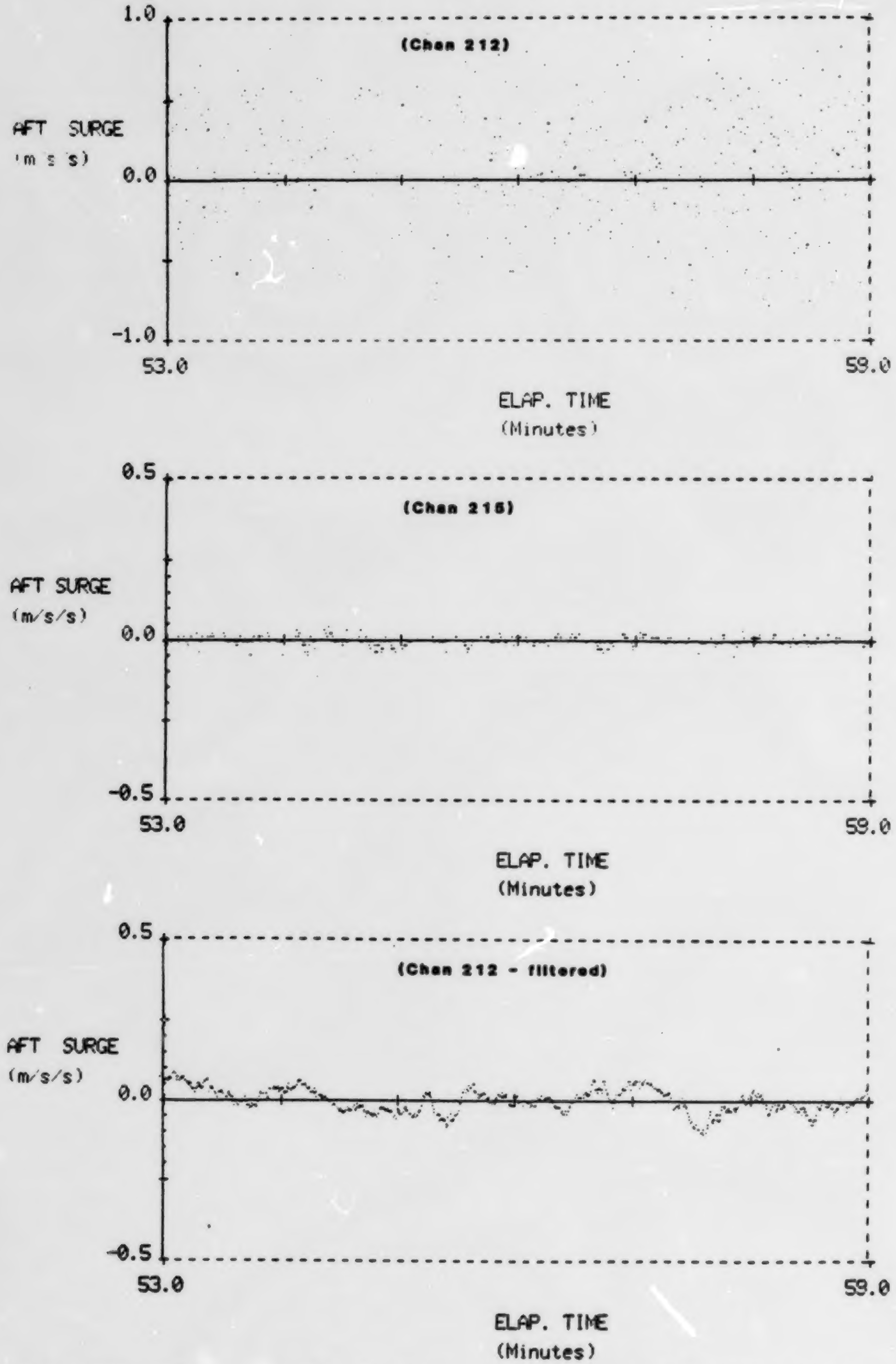


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FIGURE 11.1 Lake Melville Continuous Mode Test 12 Acceleration Plots

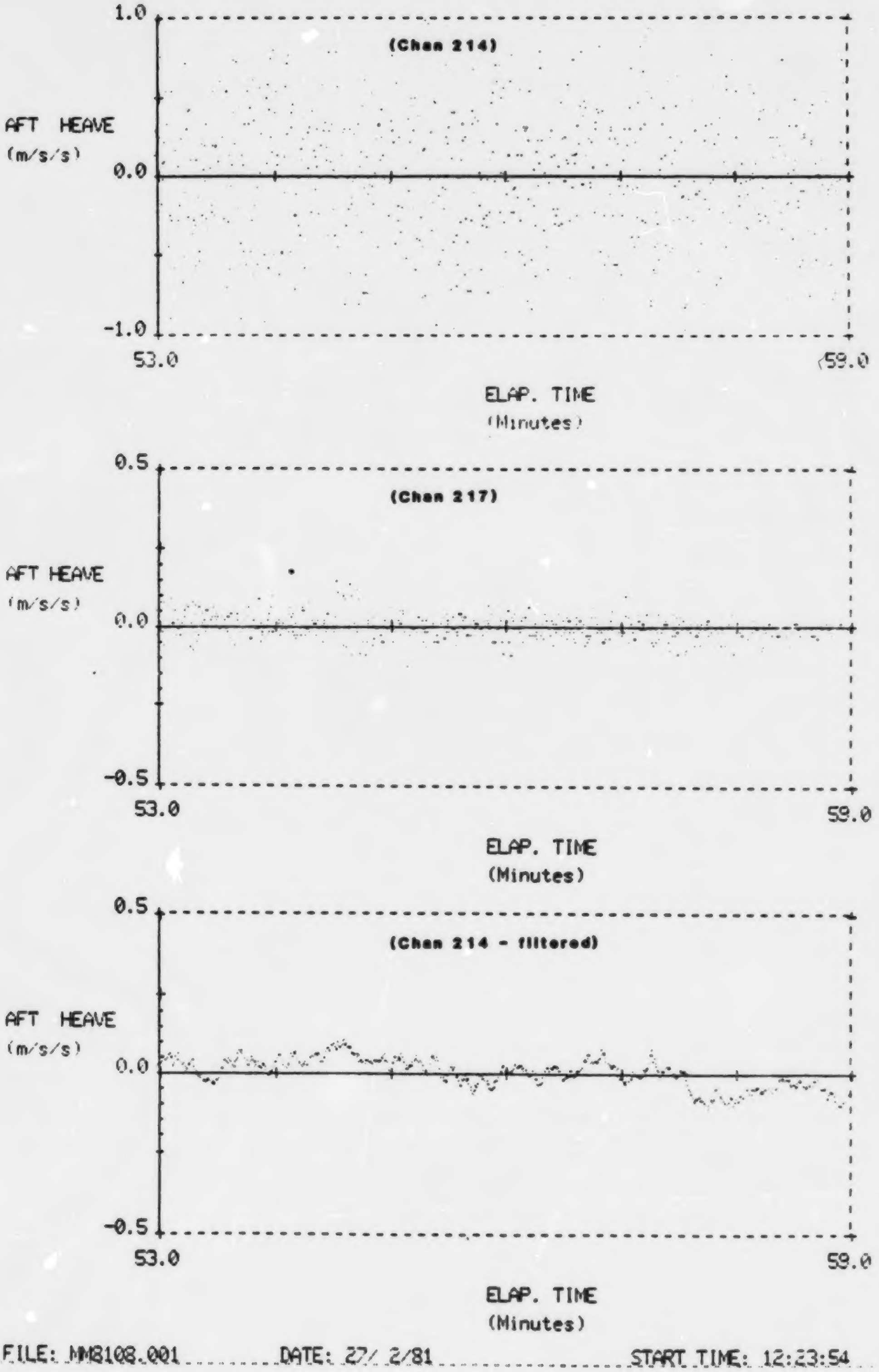

FIGURE 11.2 Lake Melville Continuous Mode Test 12 Acceleration Plots

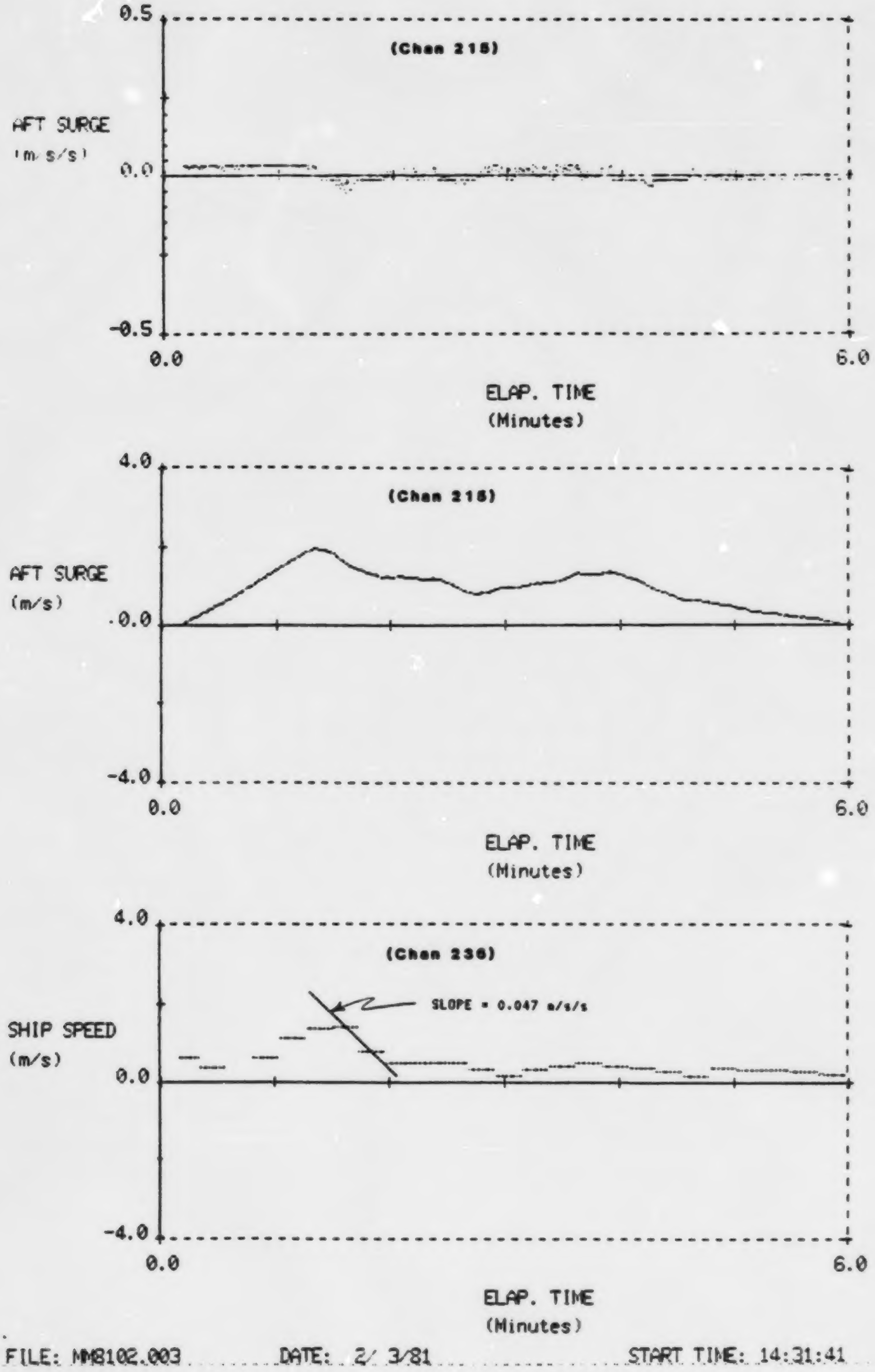

FIGURE 11.3 Lake Melville Ramming Mode Test 3 Acceleration and Velocity Plots

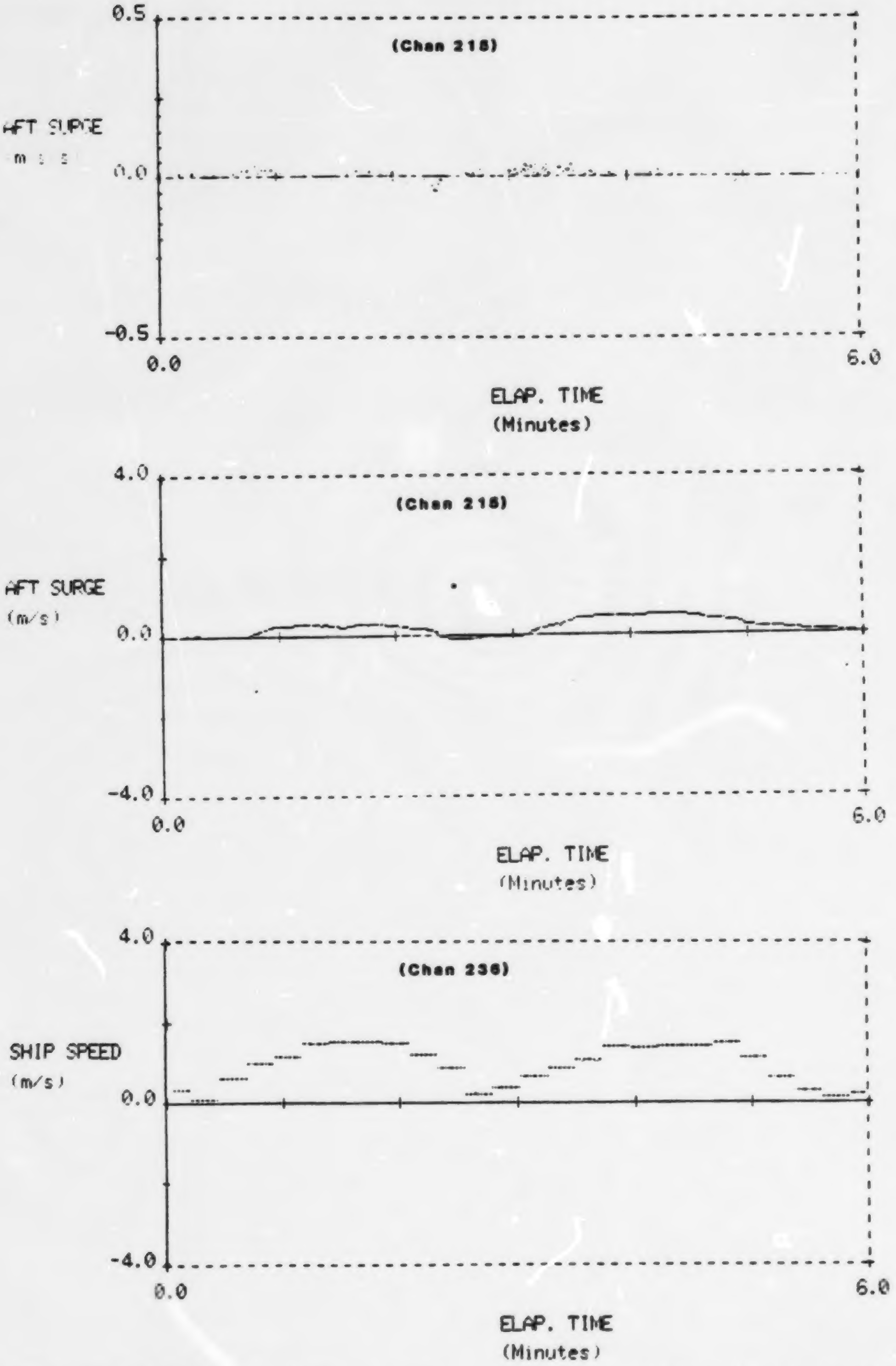

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FIGURE 11.4 Lake Melville Ramming Mode Test 6 Acceleration and Velocity Plots

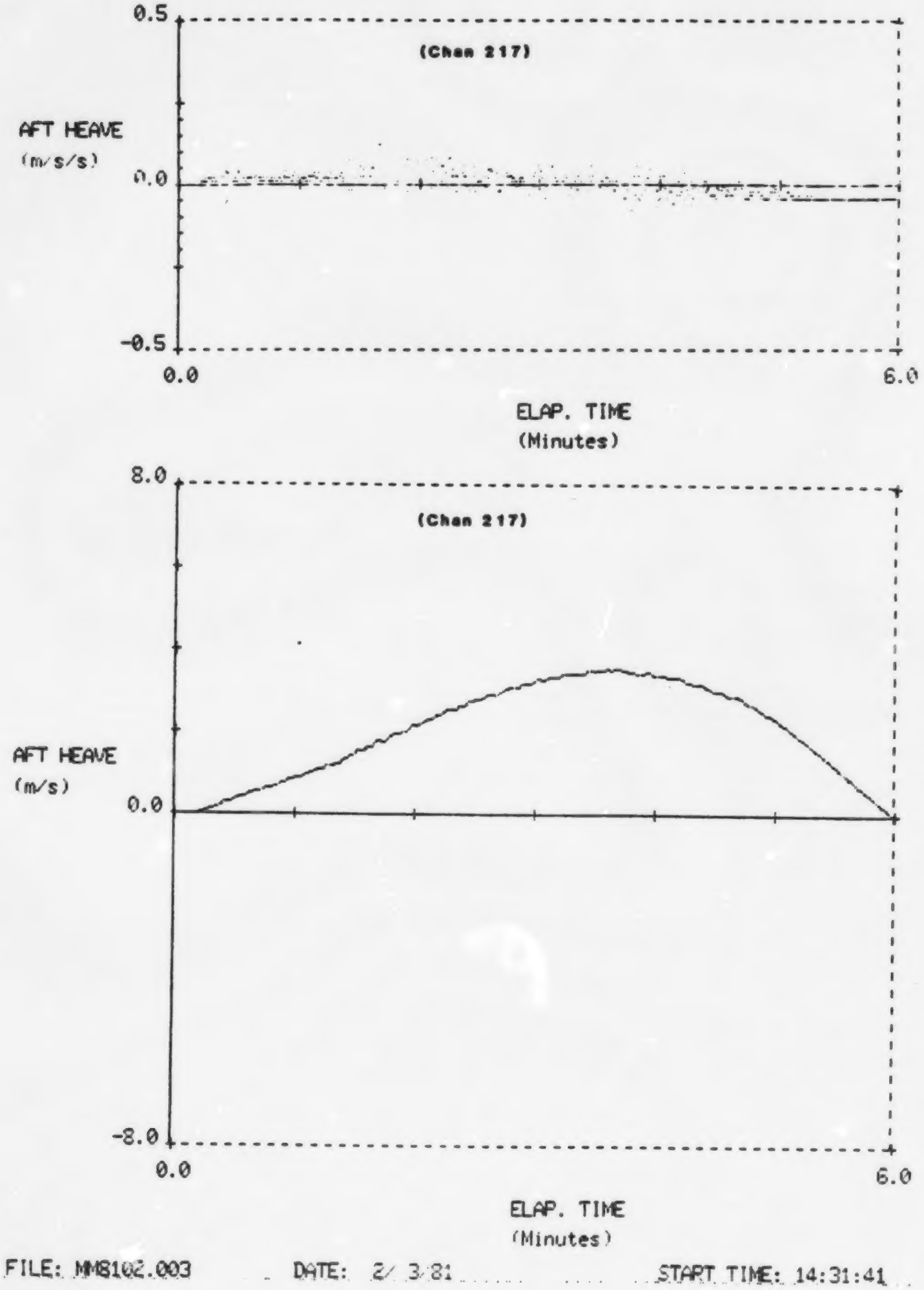

FIGURE 11.5 Lake Melville Ramming Mode Test 3 Acceleration and Velocity Plots

12.0 COMMENTARY ON INSTRUMENTATION SYSTEMS

Since this project is restricted to the analysis of the M.V. Arctic's ice transiting performance in the continuous operating mode, the comments on the instrumentation system are restricted to data relevant to analysis.

The original design of the data system appeared to be thoroughly complete, in that all the parameters one might require for a performance analysis had been included. The execution of the system was less satisfactory, however. From the very beginning of operation and for a full two years thereafter, some of the most fundamentally important sensors were operating poorly or not at all. One of the most glaring examples of this is the sensors for measuring the engine and shaft RPM, - a measurement which is generally considered to be one of the easiest to obtain with accuracy and reliability.

Part of the problem appears to have been the attitude of the equipment operators. The sensors related to ship performance were considered of secondary importance and little effort was expended to ensure that they were working reliably and accurately. The result is two years' worth of data which contains very little that is useable for a reliable assessment of the ship's performance in ice.

When the performance sensors were working properly the sampling rates and patterns were quite adequate. With the exception of the accelerations, all of the critical performance parameter recordings were found to be of good quality with little noise and good resolution. Accuracy was not always good, however, since some sensors tended to drift off calibration.

Although a more standard and convenient data tape recording format would have been desirable from a data reduction standpoint; the format used was not unreasonable or especially difficult to handle. One improvement that would have been useful is a method of verifying the data integrity such as a parity check. Far too frequently the data reduction program would encounter bad data which could not be detected and which usually caused disastrous consequences.

The data that was collected would be more useful and complete if more detailed logs of events, ice conditions and backup readings were kept. More frequent and better documented calibration checks of the data systems would also have been invaluable. In general, far more useful information would have been obtained if more short, controlled and well documented test runs were recorded rather than lengthy tapes of undocumented voyage data.

To prevent many of the aforementioned difficulties in future projects, extensive development of the data acquisition software will have to be undertaken to improve upon its utility features and error checking capabilities. Some of the features that should seriously be considered are as follows:

1. a computer based calibration manual which would interactively guide the user through a step by step calibration procedure for any selected sensor.

2. a manual mode of operation which would allow the operator to isolate a channel and assess the results for calibration checks. This would include some data conversion and graphical output.

3. as part of the data file, a configuration table which contains complete details on each channel, including declared accuracies, ranges, normal limits, calibration data, sampling rates, etc. A log of any hardware or software changes should also be included.

4. simultaneous output of related parameters graphically or otherwise during the data collection process.

5. onboard data reduction software which would allow the operators to process recorded tapes in order to improve their confidence in the data.

6. software checks which periodically test the reasonableness of the data through comparisons with limits or other parameters.

7. as an aid to the above checking process it would be useful to include some reference signals for the various classes of sensor in the data collection scan.

8. It would also be wise to include some redundancy on critical measurements relating to ship and sea conditions without which all other data becomes useless.

9. Vital information which cannot be automatically collected should be input as part of a mandatory interactive logging procedure so that it too can be included in the data file.

13.0 RECOMMENDATIONS FOR FUTURE WORK

The review and analysis of ice resistance formulations and the development of an equation to best predict the performance of the M.V. Arctic has been limited by the time and budget available for this study. The analysis has clearly shown, however, that no existing equation satisfactorily portrays the performance of the M.V. Arctic without insignificant modification.

In this report a form of the Poznyak and Ionov equation, modified in all three terms, has tentatively been adopted to represent the performance of the M.V. Arctic. It is believed, however, that further fundamental analysis and study of other test data will lead to an improved equation that will have more general application.

The following recommendations are made for further study and analysis.

13.1 Review the accuracy of full-scale and model measurements of σ_f and E with particular reference to:

(a) The length of cantilever beam in relation to the thickness of the ice.

(b) The length, breadth and orientation of the cantilever beam in relation to the size and shape of the ice pieces broken by the ship.

(c) The time period over which the cantilever beam is loaded and strain is measured compared with the time taken by the ship to break the ice.

13.2 Review in more detail the ice forces on the bow during the ice breaking process in relation to bow angles, friction, ice properties and the type of ice failure.

13.3 Review in more detail the recent work of Donald Carter[41] with particular reference to the low values of resistance predicted for the Lake Melville ice conditions and the effect of possible inaccuracies in the measurement of E as may result from item 13.1.

13.4 Study Mathew's Equation (Appendix C) and its development to include the effect of bow form as well as friction. Develop methods of calculating A, G_0 and m for various forms.

13.5 Review data from full-scale and model tests on other icebreaking vessels and develop a predictor equation which not only fits the M.V. Arctic data but also gives a reasonable fit with other full- scale data.

13.6 Review data from recent model tests on improved bow shapes for the M.V. Arctic and test ability of the derived equation to predict the effect of changes in bow shape.

13.7 Review and update the power provisions of CASPPR in the light of data from the recent model tests in Hamburg and Wartsila.

13.8 Extract from the reviews and new analyses the effects of form changes, sizes, speeds and powers, in various ice conditions, on design improvements.

13.9 Review all ice information for the various zones and seasons insofar as it affects operations.

In particular, collect together the existing data base of mechanical properties and associated measurements including on site salinities and temperatures. By statistical and other means group the information as it is relevant to level ice performance in the various zones and seasons.

Use this ice properties information and all the aforementioned studies to prepare a uniform scientific basis for design selection, operation planning and producing new regulations. Co-operate in detail with regulating bodies on this latter item.

13.10 Study data relating to the effect of internal ice pressure on resistance.

13.11 Study data relating to the effect of snow cover on resistance.

13.12 Determine how internal ice pressure and snow cover can best be taken into account in a comprehensive equation for resistance in level sheet ice. In particular, determine whether the principal effect of internal ice pressure and snow cover is on the non-velocity dependent or velocity dependent components of resistance. Use the results of this research to attempt to explain the high velocity dependent resistance to the M.V. Arctic in thin ice in Navy Board Inlet.

14.0 SUMMARY OF CONCLUSIONS

14.1 The review of existing ice resistance reasearch shows that the science of predicting the resistance of ice to the motion of a ship is still very much in its infancy. Although a great deal of data has been obtained in recent years from full-scale and model tests there is still no generally accepted equation for predicting the performance of ships of various shapes and sizes in level ice of various thicknesses and physical properties.

14.2 The data acquired over the three years 1979-81 by the instrumentation of the M.V. Arctic add greatly to the existing knowledge and has provided a factual basis for the development of a rational equation to represent the level ice performance not only of the M.V. Arctic itself but also of other ships which are not too dissimilar in shape and bow form.

14.3 The analysis and plotting of the data from the dedicated trials in continuous sheet ice indicates that the ice resistance to the M.V. Arctic is dependent on ship speed, V, and ice thickness, h, in the following manner.

(1) The velocity dependent component of ice resistance varies approximately linearly with velocity.

(2) The velocity dependent resistance also varies approximately with $h^{1.5}$.

(3) The breaking and submergence resistance not dependent on velocity varies approximately with h to the power of 1.8. In equations of the Kashtelyan type which have two non-velocity dependent terms, one varying with h and the other with h^2, this means that the h^2 term predominates.

The equations tested, in either their original or modified form, were those of Kashtelyan, Enkvist, Poznyak & Ionov and Carter. Of these, a variation of the Poznyak and Ionov equation modified in all three terms is tentatively selected as best representing the performance of the M.V. Arctic. The selected equation of total resistance (ice and water) reduces to the following when the values of hull dimensions and bow geometry are inserted for the M.V. Arctic.

$$R_t = 3.33\ \sigma_f\ h^2 + 169\ h + 241\ Vh^{3/2} + R_{ow}\ \text{(kilo Newtons)}$$

where σ_f = ice flexural strength (kilo Pascals)

h = ice thickness (meters)

V = ship speed (meters per second)

14.4 The full-power (11,000 kW) performance of the M.V. Arctic, with an Inerta 160 coating and an assumed dynamic hull/ice friction coefficient of 0.03, as given by the above modified Poznyak and Ionov equation, is shown in Figure 14.1 for an ice strength of 350 kPa. This indicates a maximum continuous mode capability of 1.0 metres of level ice at a speed of 0.8 m/s (1.6 knots).

14.5 The model test report states that "the ship will be capable of 1.5 knots through 2 ft. of 100 psi ice". This is not inconsistent with the full-scale test results if account is taken of the model assumptions of a 690 kPa ice strength, a hull/ice friction of 0.15 and a power of 12,000 H.P. (900 kW). A non-dimensional plot of the modified Poznyak and Ionov equation correlates well with the model test results if a new interpretation is given to the model friction tests as discussed in Section 8.

14.6 Theoretical considerations suggest that the bow shape of the M.V. Arctic could be improved, particularly in the area of the shoulders. Ideally the tangent of the flare angle should exceed the coefficient of friction between hull and ice over the whole area of the bow in contact with ice if the ice is to be broken by bending rather than crushing.

14.7 The bubbler system on the M.V. Arctic appears to be ineffective from the available test data. The reason for this is not known but may be due to the vertical sides of the vessel and the already low friction provided by the coating of Inerta 160. There is no evidence that any of the "bubbler on" tests were carried out in conditions of internal ice pressure and it is possible that the bubbler system would be effective under such conditions.

14.8 Performance of the M.V. Arctic in broken ice appears to conform reasonably with the predictions of Kashtelyan. However, a complete description of the broken ice environment was never recorded and a reliable indication of performance in broken ice cannot be predicted.

14.9 The M.V. Arctic test results indicate that the power provisions of CASPPR are adequate if the requirement of an Arctic Class 2 vessel is narrowly defined as its ability to transit 2 ft thick ice in the continuous mode. However, the voyage data shows that the minimum CASPPR power requirements are too low for unassisted operation of a Class 2 vessel in the ice conditions that may be met in zone 13 during the season now permitted.

14.10 As far as ice transiting performance is concerned, the instrumentation system installed in the M.V. Arctic was sufficiently comprehensive but not all sensors were functioning in the first two seasons. The data collected during voyages would have been more useful if more detailed logs of events, ice conditions and back-up readings had been kept. Some suggestions for improvement are given in Section 12.

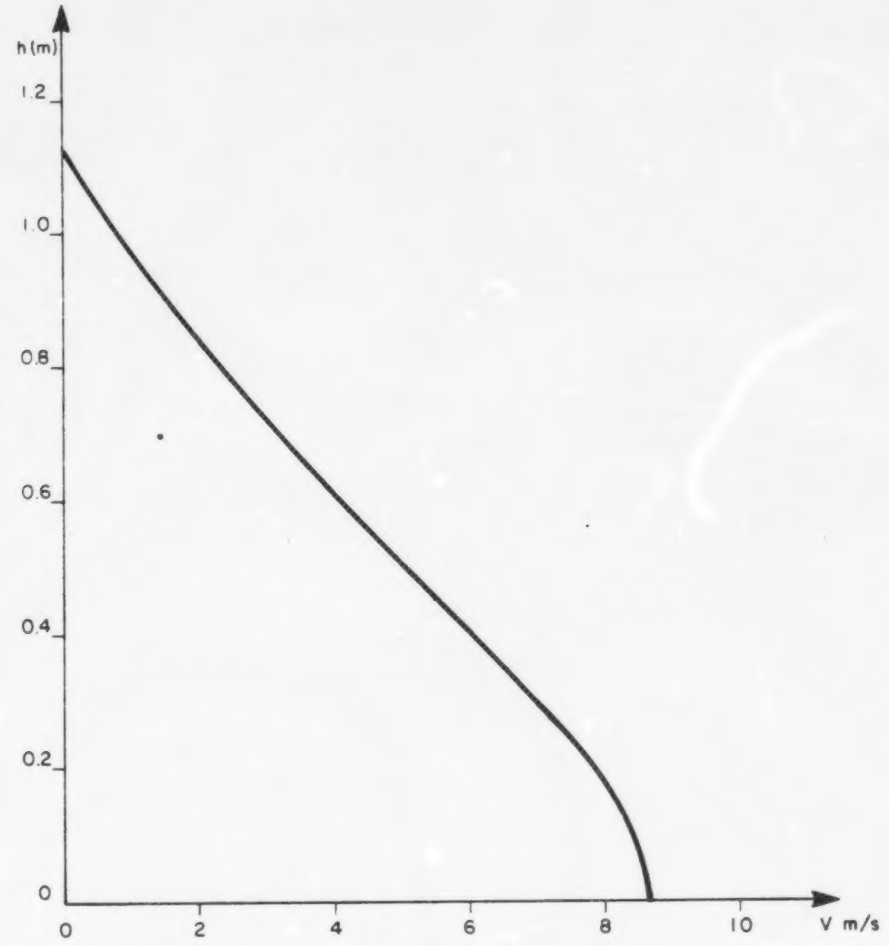

FIGURE 14.1 M.V. ARCTIC - SHEET ICE PERFORMANCE SHIP SPEED VS. ICE THICKNESS WITH FULL POWER.

σ_f = 350 kPa. f_d = 0.03

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APPENDIX A

HISTORICAL REVIEW OF ANALYTICAL AND EMPIRICAL RESEARCH RELATING TO THE RESISTANCE OF ICE TO THE MOTION OF A SHIP

1. INTRODUCTION

The accompanying report is intended to give the results of analysis of three years of recorded data from voyages and trials of the M.V. Arctic and to present equations which represent the performance of the vessel in the continuous mode when transiting ice of the following types:

(a) continuous level sheet ice
(b) broken ice in previously broken channels
(c) ice floes in various concentrations.

This Appendix presents a review of basic research and previous attempts to develop equations to predict performance in ice on an analytical or semi-empirical basis. Insofar as continuous sheet ice generally offers the greatest resistance and is more amenable to scientific study it has historically received the greatest attention. It is therefore, the resistance of continuous sheet ice which is mainly discussed.

It may be argued that a historical review of the science of predicting ice resistance to a ship is not justified and that there are already available suitable equations for predicting resistance. Unfortunately this is not so and the various equations put forward by various authorities give widely different results when extrapolated for large bulk carriers and for ice thicknesses to be encountered in the High Arctic. The predictor equations finally selected remain a matter of judgement but this historical review at least shows the uncertainties involved and helps to emphasize that any equation is based on a science that is still far from exact.

2. HISTORICAL SUMMARY

2.1 Early Work

A good description of the pioneering work of early investigators is given by Kashtelyan starting with the work of Runeberg in 1890 and continuing with the work of other Russian pioneers such as V.I. Afanasev, D.I. Mendeleev, S.O. Makaror, A.N. Krylov, Ju. A. Shimanskij, I.V. Vinogradov, V.S. Nazarov, V.L. Tsurikova, L.M. Nogid, V.I. Neganov, M.K. Tarshis and M.S. Jakovlev.

Most of the early investigators attributed the resistance to motion almost entirely to the force required to break the ice. They did, however, include in this force the fritional resistance and most of their equations included the coefficient of friction and one or more angles describing the form of the bow.

Tarshis and Jokovlev appear to be the first to divide the resistance into two components, one the ice-breaking resistance independent of velocity and the other the velocity dependant resistance of the water or ice/water mixture.

2.2 Kashtelyan[1]

The extensive analytical and semi-empirical work of Kashtelyan is still used as the basis of most of today's resistance equations. Kashtelyan in his derivation of an ice-resistance equation, makes the following basic assumptions:

- Total resistance equals direct plus velocity resistance where direct resistance includes all types of resistance not dependant on velocity; e.g. hull friction, submerging of broken ice, turning of floes, icebreaking by the ice knife and energy losses due to changes in ship position.

- The direct resistance and velocity resistance act independently.

- The icebreaker moves at a constant speed; i.e., the work done by the propellers is balanced by the work of resistance forces.

Kashtelyan's equation takes the following basic form:

$$R_t = R_b + R_s + R_v + R_{ow}$$

Resistance due to breaking of ice:

$$R_b = C_b B \sigma_f h \mu_o$$

WHERE: C_b = is a non-dimensional coefficient determined by model experiments

μ_o = an efficiency factor.

$$= 1 + \frac{\Sigma R_H}{\Sigma R_V}$$

ΣR_H = total horizontal forces

ΣR_V = total vertical forces applied on the ice by the front part of the ship.

WHEN: $\Sigma R_H = 0$, $\mu_o = 1$, there is no horizontal force and all of the energy of the propellers is converted to breaking the ice.

Resistance due to gravity dependant forces:

$$R_s = R_{s1} + R_{s2} + R_{s3}$$

WHERE: $R_{s1} = C_{s1} \cdot \frac{B}{L} h^{1.75} \sigma_f^{1.25} \mu_o$

= value of resistance caused by the change of position of the icebreaker.

C_{s1} = is a factor which takes into account all assumptions and inaccuracies allowed in the derivation of the equation.

and $R_{s2} = C_{s2} \gamma_i Bh^{1.75} E^{0.25} \mu_o$

= the resistance corresponding to turning of the ice.

C_{s2} = is a factor which takes into account all assumptions and inaccuracies allowed in the derivation of the equation.

and $R_{s3} = C_{s3} \gamma_i Bh^2 \mu_o$

= the force required to overcome buoyancy of the ice.

Kashtelyan considers R_{s_1} and R_{s2} relatively small and ignores them.

Therefore:

$$R_s = R_{s3}$$

$$R_s = C_s \gamma_i Bh^2 \mu_o$$

Where: $C_s = C_{s3}$ = non-dimensional coefficient determined by model experiments.

Resistance due to penetration through broken ice:

$$R_v = C_v B^x hv^y (1/\eta_2)$$

WHERE: C_v = a dimensional coefficient which takes into account all assumptions and inaccuracies allowed in the derivation of the equation.

η_2 = the "ice cutting coefficient" which takes into account hull shape:

$$= \frac{\Sigma R_H/\tan\alpha}{\Sigma R_H} \quad \text{(see below)}$$

x & y = constants derived from model experiments.

The final form of the equation is therefore:

$$R_t = C_B B\sigma_f h\mu_o + C_s \gamma_i Bh^2 \mu_o + C_v B^x hv^y \times \frac{1}{\eta_2} + R_{ow}$$

$$= 0.004 B\sigma h \mu_o + 3.6 \gamma_i Bh^2 \mu_o + 0.25 B^{1.65} \frac{hv}{\eta_2} + R_{ow}$$

WHERE: R_t is in tonne-metre-second units.

Kashtelyan derived the numerical coefficients for the above equation on model and full scale tests of the Russian Icebreaker 'Ermak' and, strictly speaking, these coefficients apply only to that ship. However, having incorporated parameters of icebreaker shape, i.e., η_2 and μ_o, and ship beam, Kashtelyan claims application to other icebreakers at low speeds as a means of resistance evaluation.

The hull shape coefficients μ_o and η_2 were analyzed by Shimansky, Kashtelyan and others and the following expressions in terms of the bow angles were derived:

$$\mu_o = 1 + \frac{\int_o^{L_x} (\mu_1 \tan^2 \alpha)}{\int_o^{L_x} (\mu_1 \tan\alpha \tan\beta)}$$

$$n_2 = \frac{\int_o^{L_x} (\mu_1 \tan\alpha)}{\int_o^{L_x} (\mu_1 \tan^2\alpha)}$$

WHERE: $\mu_1 = \frac{\sqrt{1 + \tan^2 \alpha}}{1 + \tan^2 \alpha + \tan^2 \beta}$

L_x = distance to position of maximum beam measured along x

x = distance from forward perpendicular along centre line of ship

α = local waterplane angle

β = local flare angle

2.3 Lewis and Edwards[2,3,4]

Lewis and Edwards have derived a number of ice resistance equations based on a series of full scale and model tests. Test information was obtained for the Soviet ships 'Vladivostok' and 'Moskva', the U.S. icebreakers 'Staten Island' and 'Mackinaw' and the Canadian icebreaker 'Louis S. St. Laurent'. Model tests have included the latter two vessels as well as the proposed icebreaker 'Polar VII'. Test results were formed and plotted in dimensionless groups so that equations developed from such plots could be compared directly with data from other ships having different dimensions and characteristics.

An equation developed from full scale data of the 'Mackinaw'[2] using a step-wise regression analysis procedure gave the following, where R_i is in tonnes and linear dimensions in feet.

$$\frac{R_i}{\rho_w g\ Bh^2} = 5.259 + 4.350\frac{v}{\sqrt{gh}} + 1.676\frac{v^2}{gh} + 5.464\frac{h_s}{h}$$

It will be noted that the ice strength does not appear in the above equation because the sea ice encountered was relatively weak and it was found that the effect of ice strength on resistance was small.

The model tests on the 'Mackinaw' gave different results from the full scale data, particularly at higher speeds with Froude numbers, $v/\sqrt{gh}$, greater than 1.5 in ft.-sec. units.

The equation derived from the model tests was:

$$\frac{R_i}{\rho_w g\ Bh^2} = 3.455 + 0.019\frac{\sigma f}{\rho_w gh} + 4.680\frac{v}{\sqrt{gh}}$$

This empirically derived solution would be applicable only when $\sigma/\rho_w gh$ lay between 30 and 356 and when $v/\sqrt{gh}$ lay between 0.02 and 2.66.

An equation derived from full scale ramming tests with the Canadian Coast Guard's 'Louis S. St. Laurent' was as follows:[3]

$$\frac{R_i}{\rho_w g\ Bh^2} = 4.242 + 0.0503\frac{\sigma}{\rho_w gh} + 8.95\frac{v}{\sqrt{gh}}$$

This equation is of the same form as that derived from model tests on the 'Mackinaw' but the velocity and ice strength coefficients are much greater.

Based on model tests of the proposed 'Polar VII' icebreaker,[3] Edwards and others derived the following equation:

$$\frac{R}{\rho_w g\ Bh^2} = 5.7 + 0.147\,\frac{H}{h} - 7.83\,C_B{}^2$$

$$+ \frac{\sigma_f}{\rho_w gh}\left[- 0.318 + 0.265f + 0.394C_B\right]$$

$$+ \frac{v}{\sqrt{gh}}\ -\left[68.16 + 0.0477\,\frac{\sigma_f}{\rho_w gh} + 223.73\,C_B - 181.3\,C_B^2 + 0.249\,\frac{H}{h}\right]$$

WHERE: C_B = block coefficient.

H = ship draft.

f = friction factor.

Apart from its complexity, the above equation differs from most previous equations in that it recognizes that draft has an effect on resistance as well as the block coefficient and hull friction, as indicated by the model tests. The equation can be applied only to the Polar VII icebreaker and to the range of variation of parameters actually tested. Some terms of the equation involve small differences between large numbers and are very sensitive to the value of C_B.

This reinforces the principle that, where regression analysis has been used, predictions should not be made outside the data base from which the equation has been derived.

2.4 Milano [5,6,7]

In 1973, Milano presented[5] a mathematical model applicable to all ships of conventional icebreaker form. The results were compared with model and full scale data and the correlation was found to be good. The equation developed considers the icebreaking energy (E_i) required to be the sum of the following components:

$$E_i = E_1 + E_2 + E_3 + E_4 + E_5$$

WHERE: E_1 = Impact of various bow and cusp wedges.
E_2 = Local crushing.
E_3 = Climb onto ice.
E_4 = Fracture and fall of ship.
E_5 = Move forward forcing ice down.

The solution to the equations are complicated and a FORTRAN program has been developed. Further valuable work[6,7] by Milano was presented in 1975 which covered resistance to continuous motion in mush ice as well as in solid unbroken ice.

2.5 E.E. Bustard[8]

Bustard developed the following equation in order to show the importance of size in an arctic ship[8].

$$SHP = Bh^2 \left\{ C_1 + C_2 \left(\frac{100V}{\sqrt{L}} \right)^n \right\}$$

WHERE: B, h, L and V are in feet units
C_1 = constant, form dependent, taken as 11.75.
C_2 = constant, form dependent, taken as 0.17.
n = exponent, taken as 1.5

His equation is a direct conversion to required horsepower and is based on work by Lewis and Edwards and Enkvist.

The following assumptions were made:

(1) The resistance R_t is composed of two basic groups of elements, each of which has several components.

(2) The first group is that necessary to overcome the initial resistance and is a function of Bh^2 denoted by R_o.

(3) The second group comprises all velocity dependent components of the total resistance denoted by R_v and is a function of $Bh^2 v^n$ where the exponent 'n' is form related.

(4) The velocity dependent resistance follows Froude's "Law of Comparison" in that:

R_v/W is equal at equivalent $V/\sqrt{L}$
and at equivalent h/H.

WHERE: R_v = velocity component of resistance (tons).
W = displacement (tons)
H = draft (ft.)

(5) There is no Reynold's effect on the frictional components.

(6) Power is proportional to resistance.

Bustard's equation is useful to demonstrate the approximate relationship betwen the size of arctic ship and its desirable horsepower. However, its assumption that ice resistance varies with h^2 is at variance with many other equations which incorporate h^2 in only one of three terms.

2.6 Kloppenburg[9]

Between 1972 and 1974 the Hamburg ship model basin tested three models of a 70,000 dwt. icebreaking cargo vessel in the continuous mode of icebreaking[9]. Analysis was carried out for three different resistance formulas, two equations put forward by Lewis and Edwards in 1970 and 1973 and one by Vance. It was found that the best fit occurred for the equation:

$$R_i = C_b \sigma_f h^2 + C_s \rho_w g\, Bh^2 + C_v \rho_w Bhv^2$$

Extensive model test data suggested the following values for the form dependent constants when the dynamic coefficient of friction was approximately 0.1 and R_i was in MPa.

$$C_b = 0.23$$
$$C_s = 8.6$$
$$C_v = 1.7$$

Kloppenburg stresses that extrapolation to other ship sizes, ice strengths and/or thicknesses can produce great errors.

Comments by Edwards on Kloppenburg's work point out that the icebreaking resistance is the time integral of the force to fail multiple pieces of ice. The first term of the equation should not therefore contain h^2 but a relationship closer to h. The omission of B, the ship's beam, from the first term was also questioned.

2.7 Mathews[10]

On the basis of work by Lewis and Edwards and others, Mathews presented the following general equation relating to ice resistance, ship beam, ice thickness, ice strength and velocity.[10]

$$R_i/\rho_w g\ Bh^2 = C_s + C_b\ \sigma_h + C_v\ F_h^2$$

WHERE: $\sigma_h = \sigma/\rho_w g\ h$ and $F_h = V/\sqrt{gh}$

While Mathews' equation is of the same form as that of Kloppenburg, the icebreaking term is dependent on B and on h rather than h^2.

The purpose of Mathews' paper was to derive a direct relationship between the size of an icebreaking ship, its arctic class or icebreaking capability and its desirable horsepower. For this purpose he assumed that there should be a minimum design speed to provide sufficient inertial reserve to allow for ridges or other non-uniformity in the ice field. For the ten existing icebreakers and the two hypothetical ships which he studied this meant a desirable ice field speed requirement up to 4.9 knots.

The resulting relationships given by Mathews are as follows:

$$\frac{R_i}{Z} = 3.76e\left[\frac{(P_s D)}{(1000Z)}\right]^{2/3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

WHERE: R_i = resistance of ice in tons.

Z = number of propellers.

e = propeller performance coefficient, 0.89 typically.

P_s = total shaft horsepower.

D = propeller diameter in feet.

$$= 8.2\left[\frac{P_s}{1000Z}\right]^{2/7} \quad \text{approximately.}$$

so that

$$\frac{R_i}{Z} = 13.6\left(\frac{P_s}{(1000Z)}\right)^{6/7} \quad \text{approximately} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

Also

$$R_i = \frac{WC_R}{1000} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

where W = displacement of ship in tons.

and C_R = resistance (thrust) coefficient.

$$= 2.86\ C_0\ (1 + yH)\ H \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where y = stopping distance parameter, taken to be 0.26.

H = $\rho gBh^2/W$, the ice thickness parameter.

C_0 = hull form coefficient, taken as 6.0

putting numerical values into equation (4):

$$C_R = 17.16\ (1 + 0.26H)\ H$$

Values of C_R are plotted against H in Mathews' paper which enable the desirable horsepower to be determined directly from equation (2) in terms of displacement and number of propellers, the beam of ship and ice thickness being taken into account in the value of C_R.

2.8 Vance[11]

In 1975, Vance published a paper[11] dealing with an ice resistance equation for continuous motion in uniform ice based on stepwise multiple regression analysis of model and full scale data of the following ships:

(1) USCGC Mackinaw
(2) USSR Moskva
(3) Tug Jelppari
(4) Ro-ro Ferry Finncarrier
(5) USCGC Staten Island
(6) USSR Ermak

Three different equations were developed with the optimum regression equation having the form:

$$R_i = C_s \rho_\Delta g B h^2 + C_b \sigma_f B h + C_v \rho_i v^2 L h^{0.65} B^{0.35}$$

The author points out that the purpose of his work is to seek a system that can be used to compare model with full scale test results. While the author claims that the resistance to a vessel progressing through ice can be predicted with reasonable accuracy, a number of discussants doubted the wisdom of extrapolation beyond the range of data analyzed.

2.9 Enkvist[12]

In a treatise published in 1972[12] Enkvist developed an equation for ice resistance based on an experimental investigation of fundamentals and the comparison of model data with full scale data for three ships breaking uniform ice in the continuous mode.

As done by Kashtelyan, Enkvist assumed that ice resistance to a ship was made up of three main components:

- the resistance due to the breaking of the ice cover.
- the resistance due to submersion of the ice.
- the velocity dependent resistance.

The formulation of Enkvist's equation is discussed below under the three main headings.

2.9.1 Resistance due to breaking ice

In order to determine the forces involved in breaking the ice, the following cases were considered:

(1) simple cantilever beam.
(2) semi-infinite beam on an elastic foundation.
(3) semi-infinite plate on an elastic foundation.

The three cases give similar parametric relationships but the numerical coefficients differ reflecting the different forces and energy required for breaking. The essential point made is that the breaking force is intermittent and that the resistance is equal to the energy absorbed in breaking divided by the distance travelled.

$$\text{i.e.,}\quad \frac{1}{2} \times \frac{\text{Maximum force x maximum deflection}}{\text{Distance travelled per breaking cycle}} = \frac{F \times \delta}{2\,\ell}$$

Since, in the case of a simple beam

$$F = \frac{\sigma B h^2}{6\,\ell} \quad \text{and} \quad \delta = \frac{F \ell^3}{3EI} = \frac{2\sigma \ell^2}{3Eh}$$

$$\text{Energy } E = \frac{1}{18}\,\frac{\sigma}{E}\,B h \sigma \ell$$

$$\text{Resistance } R = \frac{E}{\ell} = \frac{1}{18}\,\frac{\sigma}{E}\,B h \sigma$$

In other words dynamic resistance is proportional to h rather than h^2 as would be the case with static resistance.

From these considerations of the following general equation for the icebreaking resistance was proposed.

$$R_b = C_b \frac{\sigma}{E} Bh\sigma(1 + \eta_{\mu s} \times \eta_{\mu c})$$

WHERE: C_b = coefficient, dependent on assumed case of ice failure.

$$\eta_{\mu s} = \frac{1}{\sin\phi \times \cos\psi}$$

$$\eta_{\mu c} = \frac{f_d}{1 - f_d \times \frac{\sin\phi}{\cos\psi}}$$

$\sin\phi$ and $\cos\psi$ are average values integrated over the whole bow area at the water line.

ϕ = angle between stem and horizontal plane.

ψ = angle between a horizontal plane and a section perpendicular to waterline around the bow.

2.9.2 Resistance due to submersion of ice

Model tests have shown that at low speeds a considerable amount of ice rides along the bottom and sides of a hull. At high speeds the blocks of ice may be thrown down to a depth of twice the hull depth and, under these conditions, there is no independent resistance but rather a velocity dependent resistance reflected in the energy imparted to the ice. The pure potential submersion resistance is due to pushing ice with volume $Bh\ell$ and specific buoyancy $g\rho_\Delta$ down to depth ℓ. This results in an equation of the form:

$$R_s = g\rho_\Delta \ Bh \ H \left(\eta_{sp} + f_d \ \eta_{s\mu E} \ \frac{L_F}{H}\right)$$

WHERE: L_F = length of forebody.

η_{sp} = submersion ratio = $\frac{\overline{S}}{H}$

$\overline{S}$ = average depth of submersion of ice.

$\eta_{s\mu E} = \frac{A_{WLF} \times \overline{\sin \beta}}{B \ L_F}$

β = angle between section and vertical plane.

A_{WLF} = water plane area of forebody.

2.9.3 Velocity dependent resistance

Enkvist states that in many cases the dominant component of resistance is that due to velocity. The velocity dependent resistance forces investigated were as follows:

(1) simple acceleration of broken ice.
(2) friction due to dynamic pressure.
(3) turning of broken floes in the bow area subdivided into mass, ventilation and water resistance

It was concluded that the major velocity dependent components consisted of initial acceleration of broken floes:

$$R_{vm} = V^2 \ B \ (\tan^2 \phi) \ \rho_i \ (h + 0.218\ell) \ (1.0 + \eta_{\mu s} \ \eta_{\mu c})$$

and the residual turning until a floe is flat against the bow:

$$R_{vv} = B\ell g\rho_w \ (0.167\ell \ \sin\Psi + H_{BW}) \ \frac{\eta_{\mu\psi} + \eta_{\mu\theta}}{2}$$

WHERE: H_{BW} = height of bow wave = $0.5\ V^2/2g$

ℓ = length of ice beam or broken cusp.

= $5 \times h$

$$\eta_{\mu\psi} = \frac{\text{Sin}\ \phi\ \text{Cos}\ \psi\ +\ f_d\ \text{Cos}^2\ \phi}{\text{Cos}\ \psi\ \text{Cos}\ \phi\ -\ f_d\ \text{Sin}\ \phi\ \text{Cos}\ \phi}$$

$$\eta_{\mu\theta} = \text{Sin}\ \psi\ \text{Sin}\ \alpha\ +\ \text{Cos}\ \alpha$$

α = half angle of entrance of bow at waterline

The angles shown above have the meaning shown in the Glossary and are average integrated angles or the arithmetic mean over the whole bow area. The results of the analysis were then compared with model and full scale test results and it was found that correlation was poor for the breaking and submersion resistance while reasonably good for the velocity dependent forces. Based on three model test results an average correlation factor of 2 was introduced to each of the icebreaking and submergence terms.

2.9.4 Enkvist Revision 1983 [28]

In his treatise of 1972 and his resulting analytical equation, Enkvist stated and assumed that the breaking component of total ice resistance was very small. Since more recent evidence has cast serious doubts on this assumption, a number of model and full-scale tests have been carried out in pre-season ice at creeping speeds in order to determine the relative magnitude of the breaking and submersion components of resistance. The results were presented in a paper by Enkvist[28] in May 1983 and showed that the breaking resistance varies from 40 to 80 percent of the non-velocity dependent resistance, depending on the size and shape of the vessel.

Enkvist declines to present a new equation for the breaking resistance but does offer the following revised equation for the submersion resistance.

$$R_s = \rho_\Delta\ g\ [3.9\ B^2\ hf_d\ \overline{\text{Sin}\ \beta}\ +\ 12.2\ Bh^2\ \eta_{\mu\psi}]\ [N]$$

WHERE: $$\eta_{\mu\psi} = \frac{\sin\phi\cos\psi + f_d\cos^2\phi}{\cos\psi\cos\phi - f_d\sin\phi\cos\phi}$$

and $\overline{\sin\beta}$ = an integrated value of angles for the part of the forebody covered by ice.

In computing submersion resistance in accordance with the above equation, a friction coefficient of 0.08 was assumed for ships with an Inerta 160 coating in good condition and a coefficient of 0.16 for all others.

2.10 Johansson, Scarinkangas and Waltzer[13]

In addition to an analysis of the economics of winter navigation, Johansson et. al. discuss the preliminary design and model testing of the URHO-ATLE class of icebreakers.

On the assumption that the thrust of the ships should be enough to overcome the sum of open water and ice channel resistance at 4 m/s the authors propose the following formula:

$$T = C_1 \cdot C_2 \cdot C_3 + 0.017\ W^{2/3}$$

where: T = net thrust of propeller at 4 m/s in 10^4N
C1 = coefficient depending on the characteristics of the ship
C2 = coefficient depending on ice class of ship
C3 = coefficient depending on measures taken to reduce friction
W = displacement of ship in metric tonnes

Relationships and curves are given to determine C_1, C_2 and C_3 and also to obtain power from thrust according to propeller diameter.

Diesel engines were chosen in preference to gas or steam turbines because of their better fuel economy and four alternative transmission systems were considered.

(1) 4 screws, DC-DC
(2) 4 screws, AC-DC
(3) 4 CP screws, mechanical
(4) 2 CP screws, mechanical and air bubbling system

Alternatives (1) and (4) were selected for final testing. The model tests showed that both alternatives were completely adequate in Baltic ice up to one metre thick. The four-screw icebreaker was chosen, however, because of its better manoeuvrability and better performance through ridges which it could traverse continuously without ramming for ridge heights from 8.5 m to 11.0 m depending on friction.

2.11 Makinen, Lahti and Rimppi[14]

Makinen et. al. investigated the hull surface friction on ice resistance both in model tests and full-scale tests. They showed friction to be of dominant significance and involved in all components of ice resistance.

For the icebreakers tested (Jelppari, Valpas and Silma) the level ice model resistance increased by a factor of four as the friction coefficient varied from 0.05 to 0.32. The friction also varied with the quality of the ice and snow surfaces and the type of ice-hull contact.

2.12 Vance, Goodwin and Gracewski[15]

Vance et. al. presented the results of full-scale tests of the icebreaking tug Katmai Bay in Lake Superior and the St. Mary's River. The principal characteristics of the vessel were:

Length, overall	42.7 m
Maximum beam	11.4 m
Maximum beam at DWL	10.4 m
Mean draft	3.7 m
Maximum displacement	660 tonnes
Shaft Horsepower	2,500
Single propeller	

Equations developed by regression analysis for tests in level ice were as follows:

(1) Continuous level ice:

$$R_i = C_1 \rho_\Delta g Bh^2 + C_2 \sigma_f \frac{V}{g^{0.5}} Bh^{0.5}$$

(2) Brash ice:

$$R_i = C_1 \rho_\Delta g Bh^2 + C_2 \rho_i V^2 LB^{0.35} h^{0.65}$$

The authors claim that the use of the bubbler system reduced the power requirement some 25 to 35% in level ice and some 30 to 50% in brash ice. The respective coefficients were as follows:

	C_1	C_2
Level ice, bubblers off	55.84	0.0188
Level ice, bubblers on	46.79	0.0208
Brash ice, bubblers off	4.412	0.0306
Brash ice, bubblers on	4.011	0.0278

It should be noted that resistance of level ice indicates a linear relationship with V while the resistance of brash ice varies with V^2. It should also be noted that the breaking resistance is combined in the same term as the velocity resistance which is unusual and differs from the equation developed by Vance from previous full-scale tests[11].

2.13 Edwards, Dunne, Comfort, Bulat and Johnson[16]

The authors' paper describes the results of extensive trials of the Canadian Coast Guard ship Pierre Radisson in Arctic ice during July and August 1978, and in the Saguenay River and St. Lawrence River and Gulf in February 1979. The important vessel characteristics are as follows:

	Deep	River
Length, overall	98.35	96.5
Length, B.P.,	87.95	86.11
Beam,	19.51	19.22
Draft	7.16	6.08
Gross Tonnage	5,911	
Shaft Horsepower	13,600 10,130 kW	
Screws	2	
Propulsion	Diesel Electric, AC-DC	
Flare angle at bow	50°	64°
Stem angle	18°	18°
Friction Factors, f	0.30 ± 0.07 (low speed) 0.19 ± 0.12 (high speed)	

Due to the ice conditions available during the test period, a satisfactory characterization of the ship's performance was possible only in level ice of two dissimilar types:

- Arctic summer ice, weak, thick melting one-year ice
- Winter fresh and saline river ice.

The authors draw the following conclusions from the full scale tests:

1. Propeller thrust increases linearly with ship speed up to 10 knots for ice thicknesses between 0.25 and 0.75 m.

2. Propeller thrust also varies linearly with ice thickness within the range of thickness tested.

3. Snow cover increases resistance significantly.

4. The thrust required to commence motion is much higher than that required to maintain very low forward speed.

5. The apparent influence of ice strength is small.

6. Comparing full-scale results with model tests[17], the latter appear to over estimate resistance at low speeds and to agree well at speeds above 4 knots.

2.14 Noble and Bulat[17]

Noble and Bulat conducted tests on four different hull shapes for the Canadian Coast Guard R-Class icebreakers Pierre Radisson and Franklin. Only the shape was altered and all other characteristics such as length, beam, draft and displacement remained the same.

Hull No. 1 was the original vessel form with a mid-ship's flare angle of 5.5°.

Hull No. 2 had a mid-ship's flare of 12° which was evenly flared into the original parent bow lines.

Hull No. 3 had a mid-ship's flare of 12° which was gradually increased at each section of the whole forward half of the vessel.

Hull No. 4 was essentially the original hull with a knuckle added to assure that the ice would be forced down and broken by a surface with a large flare angle before reaching the shoulders of the vessel.

The model tests indicated that, in relation to the original hull:

- Hull No. 2 was a slight improvement
- Hull Nos. 3 and 4 were approximately equal in performance and had significantly less resistance.

The authors assumed that the velocity component of the resistance varied linearly with V. However, the actual plot of results given in the paper does not convincingly show this. It does indicate, however, that the improved bow shapes have a greater effect at high speeds (5 m/s) and larger ice thickness.

The authors assumed that resistance could be given by the following general equation:

$$R_i = C_1 h^{n_1} + C_2 h^{n_2} V$$

From the various tests and the use of regression analysis, the authors obtained the following values for the coefficients and exponents:

Hull No.	C_1	n_1	C_2	n_2
1	0.723	1.66	0.4751	1.82
2	1.329	1.75	0.1915	1.45
3	0.764	1.83	0.2282	1.07
4	0.757	2.3	0.2246	0.79

It should be noted that the derived exponents, n_1 and n_2 vary with each hull which suggests that there is something fundamentally inadequate about the assumed equation as a means of predicting results.

2.15 Michailidis and Murdey[18]

The authors present data from two probes into Lake Melville by CCGS Franklin, whose main characteristics are as follows:

L_{WL}	93.00 m
L_{PP}	87.94 m
Beam, B	19.37 m
Draft, T (mid-ships)	6.93 m
Displacement	7,825 tonnes
Propellers	2 x 4.12 m diam.

Five sets of dedicated runs were made as follows:

(a) ship in a broken channel
(b) ship in level ice nominally 0.65 m thick
(c) manoeuvring in level ice
(d) ship in open water
(e) ship in level ice nominally 0.80 m thick

Measured values of shaft power, shaft RPM and shaft thrust are plotted against ship speed and compared with predictions from model tests[17]. Discrepancies exist which highlight the need for improved prediction techniques.

Measurements of the flexural strength of the ice, σ_f, and the Modulus of elasticity, E, obtained by breaking cantilever beams, gave results of 330 kPa and 2,450 MPa respectively.

2.16 Kramek and Gulick[19]

The authors describe the problems of the U.S.C.G. Polar Class icebreakers since delivery in 1976. These problems relate mainly to the inadequacy of the strength of the controllable reversible pitch propellers and the longitudinal shaft vibrations. However, with respect to overall performance in ice, the authors suggest that the more efficient "White" bow yields large blocks of ice which interfere with and damage the propulsion system. The bow design must therefore be integrated with the design of the propulsion system to optimize overall efficiency and reliability.

2.17 McIntosh and Welsh[20]

The authors discuss the instrumentation of the USCGC Polar Star for full scale ice-breaking tests and present the results of selected portions of data obtained during Antarctic tests. The particulars of the vessel are:

Length, overall	121.6 m
Length, at 9.1 m WL	110.9 m

Beam, max. at 9.1 m WL	24.1 m
Draft while ice-breaking	9.1 m
Displacement at 9.1 m WL	12,395 t
Triple, controllable pitch propellers	
Max. power per shaft, diesel electric	4,474 kW
Max. power per shaft, turbine	14,914 kW

The ice properties which were not recorded coincident with the machinery data, were:

	26 Dec./77	28 Dec./77	29 Dec./77
Type	Pack	Fast	Fast
Ice Thickness, m	1.4	1.73	2.22
Snow Thickness, m	0.24	0.40	0.44
Ice specific gravity	0.90	0.90	0.86
Flexural strength, kPa	249	249	263
Compressive strength, kPa	733	862	790

The calculated resistance ranged from 1580 to 2950 kN from torque data and from 1780 to 3770 from thrust loadcall data. It was stated that the results indicate that the ice-breaking resistance of the Polar Star is close to that predicted by Milano's latest (1980) mathematical model and lower than that predicted by Arctec's model tests.

2.18 Schwartz, Jochmann and Hoffman[21]

The authors describe the technological aspects of a proposed German polar research vessel and the results of model tests carried out at the Hamburg Ship Model Basin. The ship will be built according to ARC 3 to break 1.0 m thick level ice in the continuous mode and has the following specifications:

Length, L_{pp}	102.20 m
Beam, max.	25.00 m
Draft, max.	10.50 m
Shaft Power	2 x 6,420 kW
Propellers	2 ducted, 4.20 m diam.

A feature of the design is that the forebody frames are concave up to CWL so that the ice is broken in bending by relatively flat flare angles and compression of the ice in the shoulder area of the forebody is avoided. The stem angle is 22.5° and the angle of waterline entrance is 28°. The lower, underwater part of the forebody is steep to push the ice aside and prevent it passing under the ship towards the propellers.

One of the main features of the Hamburg model tests is the correction made to overcome the fact that it is impossible to simultaneously obtain the correct scale for σ_f and E with saline ice. The breaking component of ice resistance is given by:

$$R_b = C_b \frac{\sigma_f^2 h^2}{E} = C_b \, \sigma_f h^2 \times \frac{\sigma_f}{E}$$

In the Hamburg tests the model ice was made over strength at 60 kPa in order to keep the ratio σ_f/E the same in the model as in full scale. A correction was then made from the resistance results to compensate for the higher strength. In the self-propulsion tests the correction was made by reducing the ice thickness.

No predictor equation is given in the paper but it is claimed that the German Polar Research Vessel will transit 1.0 m thick ice of 500 kPa strength at 5.2 knots and 1.35 m thick ice at 1.0 knot.

2.19 Narita and Yamaguchi[22]

The authors describe some experimental studies on hull forms for the new Japanese Antarctic Research Ship. Twin-screw and triple-screw icebreakers were modelled with main particulars as follows:

	Twin-Screw	Triple-Screw
Length, waterline	105.0 m	124.0 m
Breadth at design draft	24.0 m	27.0 m
Design draft	9.0 m	9.25 m
Displacement	12,000 t	17,600 t
Propellers, Number	2	3
Propellers, Diameter	5.0 m	4.9 m
Engines, type	diesel-electric	
Engines, output	2 x 12,000 shp	3 x 10,000 shp

The icebreaking model tests were carried out at HSVA in accordance with the usual procedures adopted in Hamburg. One key procedure is to maintain a constant ratio for E/σ_f of about 2000. In order to do this, the strength of the model ice is about twice what it should be for correct dimensional similarity. Adjustments to the measured resistance are made subsequently. (In self-propelled test the thickness of the ice is reduced to compensate for the extra strength.)

The main findings of the model tests were:

1. Of the four hull forms tested, the "cylindrical" stem on Model C was the best both in the continuous and ramming mode and was observed to break ice more by bending than by crushing, compared with Model A which had the typical features of earlier icebreakers.

2. The resistance was roughly proportional to the square of the ice thickness.

3. The effect of the ice flexural strength on the icebreaking resistance was considerable.

4. The resistance increased substantially (about 75%) as the hull-ice friction coefficient increased from 0.1 to 0.2.

Finding number 3 is different from that of many other researchers who have tended to find that the resistance is not greatly affected by the ice flexural strength. This may be at least partly due to the modelling procedure of using a model strength twice that required by correct scaling laws and making a subsequent adjustment. In the discussion on the paper two speakers cast considerable doubt on the procedure used for making the correction.

2.20 Freitas[23]

The author describes the experimental investigation of a fundamentally new concept of icebreaker form being developed in collaboration with the Hamburg Ship Model Basin (HSVA) and known as the Waas concept after its originator Dr. H. Waas.

The essential elements of the Waas concept are:

- a hull with a basically square bow, a flat sloping bottom and vertical sides parallel to the centre line.

- a bow width greater than the beam of the ship with runners along the edges of the bottom to perform as shearing edges.

- a gradual flaring of the flat bottom bow into a V-shape and then into the mainbody of the ship.

The main advantages claimed for the Waas concept are that the ice is broken entirely by shearing and cantilever bending and there is no crushing of the ice as occurs at the shoulders of the conventional ship form. It is also claimed that the greater part of the ice is displaced under the ice cover, thus reducing friction betwen the ice and the ship's hull and icemilling in the propellers.

Three vessels were tested with characteristics as follows:

Characteristic		LNG Tanker	Polar Research Vessel	Pusher Tug
Length OA	m	304.0	113.0	22.0
Length BP	m	270.0	98.6	20.0
Beam, midship,	m	39.0	24.0	7.0
Beam, bow	m	43.0	-	-
Draft,	m	12.7	10.0	1.5
Displacement	t	105,000	14,100	135
Model Scale		1:36	1:16.8	1:8

In order to obtain adequate elasticity in the model ice and an E/σ_f ratio close to that of natural sea ice, the tests were done in ice with a strength of 60 kPa, higher than the correct scaled value. The measured ice resistance was then corrected by a linear extrapolation of the ice resistance as a function of ice strength, based on the following definition of pure breaking resistance:

$$R_B = C_B \frac{\sigma_f^2 \cdot h^2}{E} = C_B \, \sigma_f \, h^2 \cdot \frac{\sigma_f}{E}$$

and assuming that σ_f/E is constant.

At the time of publication of Freitas' paper (1981) prototype and model tests were being carried out on an existing icebreaker of 51 m length and 2280 kW power fitted with a Waas bow. Comparative model tests gave the following results, using a model scale of 1:10, an ice thickness of 5 cm and a flexural strength of 60 kPa.

MODEL						FULL SCALE	
Model Form	Moment Nm.	Thrust N.	Revs. 1/s	Power W	Speed m/s	Power kW	Speed m/s
Existing	13.19	171.1	8.45	700	0.51	2210	1.60
Waas 1	14.08	193.8	8.77	780	0.96	2470	3.04
Waas 2	12.82	197.8	8.32	670	0.77	2120	2.42

The author claims that the model tests have established the superiority of the Waas concept and also given a better understanding of the Waas icebreaking technique.

2.21 Poznyak and Ionov[24]

This important paper describes the results of theoretical and experimental studies for estimating the components of ice resistance including the frictional component in addition to the customary components of breaking, submergence and velocity.

For the particular model icebreaker tested, having a friction coefficient of 0.08, the components of model ice resistance were:

- Resistance associated with elastic icebreaking forces, R_b, 41.3%
- Resistance associated with friction between ship's hull and ice, R_f, 30.0%
- Resistance due to the velocity of the icebreaker at a model velocity of 0.13 m/s, R_v, 12.9%
- Resistance due to the submerging of the ice, R_s, 15.9%.

In the analytical model, based on the physical model test results, the hull friction was assumed to contribute to each of three main components of ice resistance so that:

$$R_{ice} = R_b + R_{fb} + R_s + R_{fs} + R_v + R_{fv}$$

where the interaction terms with f suffixes are resistance associated with external friction.

The following general formula for the resistance to a ship in ice was derived:

$$R_i = 0.014\, \sigma_f\, h^2 \left[a_1 (\beta)\, B + f_d\, a_2 (\beta)\, L\right]$$

$$+ 0.1 \gamma_\Delta h\, BL \frac{(\tan\alpha_o)}{(\tan\alpha_o + B/L)} \left[a_1 (\beta) \sin\alpha_o + f_d a_2 (\beta) (1 + \cos\alpha_o)\right]$$

$$+ 1.5\, \gamma_{ice}\, h\, B^2 F_L \left(1 + \frac{1}{\cos\alpha_o}\right) \left[\frac{(\tan\alpha_o)^2}{2 \tan\alpha_o - B/L} + f_d\right]$$

WHERE: F_L = Froude number = $\frac{V}{\sqrt{gL}}$

and $a_1(\beta)$ and $a_2(\beta)$ are non-dimensional coefficients of hull shape, derived from the following equations:

$$a_1 (\beta) = \frac{1}{n} \Sigma_o^n \frac{0.57}{\sin\beta} (1.6 \cos\beta + 0.11)$$

$$a_2 (\beta) = \frac{1}{n} \Sigma_o^n \frac{1}{\sin\beta}$$

The authors claim that their analytical equation has given estimates on ice resistance which are in good agreement with full scale trials and model experiments.

2.22 Kannari[34]

The author describes full-scale measurements of the performance of a 16000 dwt. icegoing product tanker in ice-clogged channels formed by constant traffic. The main findings are as follows:

1. In an ice field with general thickness of 0.75 m, brash ice accumulates over the whole channel and ridge-like keels are formed on each side of the channel to a depth of about 3.0 m. The effective breadth is considered to be 1.2 times the beam of the vessel and the mean thickness over the effective breadth varied from 1.01 to 1.22 m for three different channels.

2. The build-up of brash ice is not necessarily symmetrical about the centre line of the channel and variations also occur longitudinally.

3. Block size is affected by degree-days and density of traffic and mainly varies from 0.4 to 1.2 metres.

4. Friction resistance plays an important role in total channel resistance.

5. The net ice resistance in the channel is independent of velocity but sensitive to changes in draft. For instance, net ice resistance varied from 120 kN with a draft of 6 m to 360 kN with a draft of 9.6 m.

2.23 Kashtelyan (1983)[35]

In this paper Kashtelyan evaluates the scaling effects in icebreaking model tests, in particular those relating to the impossibility of simultaneously obtaining the correct scale for σ_f and E when using ice of high salinity. However, tests made in the AARI Ice Facility have indicated that the requirements for effective similarity can be met if the model tests are run in ice whose strength is higher than that needed for rigorous similarity. The procedure for doing this is discussed.

2.24 Nakamura et al.[37]

The authors describe 1:40 scale model tests on a high arctic class 200,000 dwt tanker using carbamide ice which they claim satisfies the similarity laws for ice modelling.

The tanker has a spoon bow with reamer, a twin-skeg stern with two ducted propellers and two rudders. The full-scale dimensions are:

Length PP,	m	360
Beam, maximum,	m	57
Draft,	m	52
Block coefficent		0.746
Shaft horsepower		150,000

The tested ice thickness was from 1.22 to 3.05 metres and the standard ice flexural strength adopted was 500 kPa. 1000 and 1500 kPa were also used for comparison in extremely hard ice. The main results were as follows:

(a) Icebreaking resistance increases almost linearly with ship speed and a "Milano hump" does not appear.

(b) Icebreaking resistance at zero speed is about 400 tonnes and at 6 knots is about 700 tonnes, which is 30% smaller than that of the conventional bow. Icebreaking by the spoon bow was observed to be very smooth and all in bending, which is considered to be the reason for its small resistance.

(c) The aftbody resistance is about 15% of the total resistance, which is unexpectedly small. Therefore, the importance of the bow part design was recognized again, because about 85% of the total resistance is caused by the forebody.

(d) The open water resistance obtained by multiplying the model value by λ^3 is 9 - 16 % of the total resistance in the speed range of 5 - 8 knots.

(e) Icebreaking resistance at a speed of 3 knots increased slightly faster than the linear power of h, - from 300 tonnes at h = 1.52 m to 720 tonnes at h = 3.05 m. Icebreaking resistance at 6 knots in 2.44 m ice increased almost linearly with ice flexural strength from 700 tonnes for σ_f = 500 kPa to 1050 tonnes for σ_f = 1500 kPa.

(f) As compared with tank test values, Enkvist's equation and Dorgeloh's equation give reasonable values, but Kashtelyan's equation gives extremely large values.

The main conclusions from the tests were:

(1) The arctic tanker with spoon bow and two-shaft stern is excellent from the viewpoint of the total performance in ice sea and open sea.

(2) The required horsepower in 2.44 m thick level ice is about 80,000 PS at V_s = 6 kts, which is much smaller than expected.

(3) The icebreaking performance of the spoon bow is superior to that of the conventional bow.

2.25 Carter[41]

In March 1983 Donald Carter presented to the Transportation Development Centre a report entitled "Ship Resistance to Continuous Motion in Level Ice". Carter's report is particularly interesting because it offers a new analytical approach to the assessment of the resistance of sheet ice to the motion of a ship instead of one based on the theories of Kashtelyan.

In essence Carter analyzes the quasi-static resistance involved in breaking the ice and then modifies this to obtain the dynamic resistance by introducing the effect of the upward pressure of the bow wave under the ice.

Carter analyzes the geometry of ice-breaking in detail and divides the quasi-static resistance, R_{is}, into two components, R_1, the resistance referred to the stem and R_2, the resistance referred to the sides of the bow. The ice is assumed to break in bending or buckling depending on the ratio of vertical to horizontal forces.

The equation derived by Carter is as follows:

$$R_t = R_{is}\left[1 + \frac{0.4\ V^2\ \ell^2}{\sigma_f\ h^2}\right]^{\frac{1}{2}} x \left[\frac{\sigma f + \sigma_o}{\sigma_f}\right] + \sigma_o h\ (2L - L_b)\ f_d + 900h_s + R_{ow}$$

WHERE: $R_{is} = R_1 + R_2$

$$\text{and}\quad R_1 = 0.548\left[\frac{\pi}{2} - \alpha_o\right] x \left[\frac{\sin\alpha_o\ \sin\phi\ + f_d\ \cos\phi}{\sin\alpha_o\ \cos\phi\ - f_d\ \sin\phi}\right]\sigma_f\ h^2$$

$$\text{and}\quad R_2 = 0.75\ \sigma_f\ h^2\ \frac{B}{\ell}\left[\frac{1 + 2\cos\phi}{(\sin\alpha_o + \cos\alpha_o)\ (1 + 2\sin\beta_o)}\right.$$

$$\left. x \left[\tan\phi + f_d\left[\frac{L_b}{B} \cdot \frac{2}{1 + 2\cos\phi} + 1.73\tan\phi\right]\right]\right]$$

WHERE: ℓ = characteristic length of broken ice plate

$$= \left[\frac{Eh^3}{12\ \gamma_w\ (1 - \nu^2)} \right]^{1/4}$$

L_b = twice the bow waterline length from stem to maximum beam.

σ_o = effective internal ice pressure

and other symbols are as given in the glossary of symbols.

(It should be noted that the symbols for the bow angles are different from those used in Carter's report.)

The terms containing σ_o and h_s in the general equation represent the effect of internal pressure in the ice cover and snow cover. If there is no internal pressure and if the effect of snow cover is accounted for by adjusting the coefficient of friction, the general equation reduces to:

$$R_t = R_{is} \left[1 + \frac{0.4\ V^2\ \ell^2}{\sigma h^2} \right]^{1/2} + 900\ h_s + R_{ow}$$

WHERE: $R_{is} = R_1 + R_2$ as before

Features in the Carter equation that appeal to the writer of this review are:

- the introduction of ice sheet failure by buckling instead of the usual assumption of failure in pure bending
- the use of the effect of the bow wave to compute what is probably the main part of the velocity dependent resistance.

However, assumptions that the writer questions pending further consideration are:

(1) That, for a general predictor equation, as opposed to one giving maximum quasi-static resistance, the average breaking resistance should be assumed to depend on quasi-static forces rather than the time integral of a series of energy expenditures divided by distance. This assumption makes the breaking resistance dependent on h^2 rather than h.

(2) That the ice breaks in a series of truncated wedges along the sides of the bow rather than in the form of cusps.

(3) That the uplifting pressure due to the bow wave keeps its maximum value over the entire length of the bow section. This seems unlikely on the basis of the hydraulic flow pattern , round the bow of a ship.

(4) That the resistance due to submersion of the ice is negligible. Most researchers in the past have considered this resistance to be significant and recent tests described by Enkvist[28] indicate that the submersion resistance is from 20% to 60% of the total non-velocity dependent resistance.

2.26 Tsoy[38]

The author describes full-scale trials to determine the resistance to ships following a channel broken by an icebreaker and presents a diagram for obtaining the resistance in terms of relative speed, V_r, relative ice thickness, h_r, and relative beam, B_r, where:

V_r = ratio of speed in channel/speed in open water
h_r = ratio of thickness of ice sheet/maximum continuous ice thickness ship can transit at 2 knots
B_r = ratio of ship's beam/breadth of channel broken

The diagram can be presented analytically as follows:

$$V_i = V_o \left[1 - K_{ch} \frac{h\,(V_o - 2)}{h_i\, V_o} \right] \quad \text{in knots}$$

WHERE:

V_i = speed in channel in knots

V_o = speed in open water in knots

h = channel edge ice thickness in metres

h_i = thickness of level ice through which ship can move continuously at 2 knots

K_{ch} = coefficient accounting for the width of the channel

and K_{ch} = $0.116 + 0.28B_r^{2.29}$ for $B_r < 1.0$

= $0.39 + 0.52\,(B_r - 1)$ for B_r between 1.0 and 1.4

= $1.0 - 0.56/B_r$ for $B_r > 1.4$

To apply the above formula it is necessary to know V_o and h_i. In case these are not known, approximate expressions determining these parameters are given.

The full-scale tests were carried out with the three icebreakers Semyon Deznev, Murmansk and Krasnin whose beams on the design waterline are 17.5, 23.5 and 25.6 metres respectively. The propulsion capabilities of the middle size icebreaker, Murmansk, in the narrower and wider channels formed by the other icebreakers are given below in terms of the ratio of power requirement to power output at same speed in open water.

Condition	Speed m/s	
	2	4
Continuous Ice	10.2	5.3
Narrow Channel	4.75	2.88
Own Channel	3.25	2.72
Wide Channel	1.40	1.38

The above table clearly indicates the effect of the width of the channel which could be greater in the case of cargo vessels with little or no side flare.

2.27 Kashtelyan and Tsoy[44]

The authors describe full scale and model tests on the problems associated with the passage of a ship through a channel made by an icebreaker with particular reference to the relative beams of the icebreakers and following ship. Little data is given but the following general conclusions are made:

1. The performance of cargo carriers following icebreakers can be estimated when the ship's open water speed and its limiting ice thickness for continuous motion in level ice is known.
2. A typical icebreaker with 18 degree mid-section flare can widen a channel 30 percent narrower than her own breadth in ice twice as thick as the limiting continuous ice cover.
3. A cargo ship with a mid-section flare of 10 degrees can move in a channel 15 percent narrower than her breadth in ice 1.5 times as thick as the limiting continuous ice cover.

No indication is given of the effect of friction or of the performance of a cargo vessel with no mid-section flare.

2.28 White

Commander R.M. White describes an analytical method for predicting the performance of an icebreaker when ramming, when operating in the continuous mode and when extracting itself by going astern after being brought to a halt. The analysis, which is capable of giving quantitative results for a particular vessel, indicates that the following characteristics are desirable for operations as follows:-

For ramming:- large displacement; high impact velocity; small bow angle at waterline; small spread angle complement; low kinetic friction; high waterplane coefficient; high beam to draft ratio.

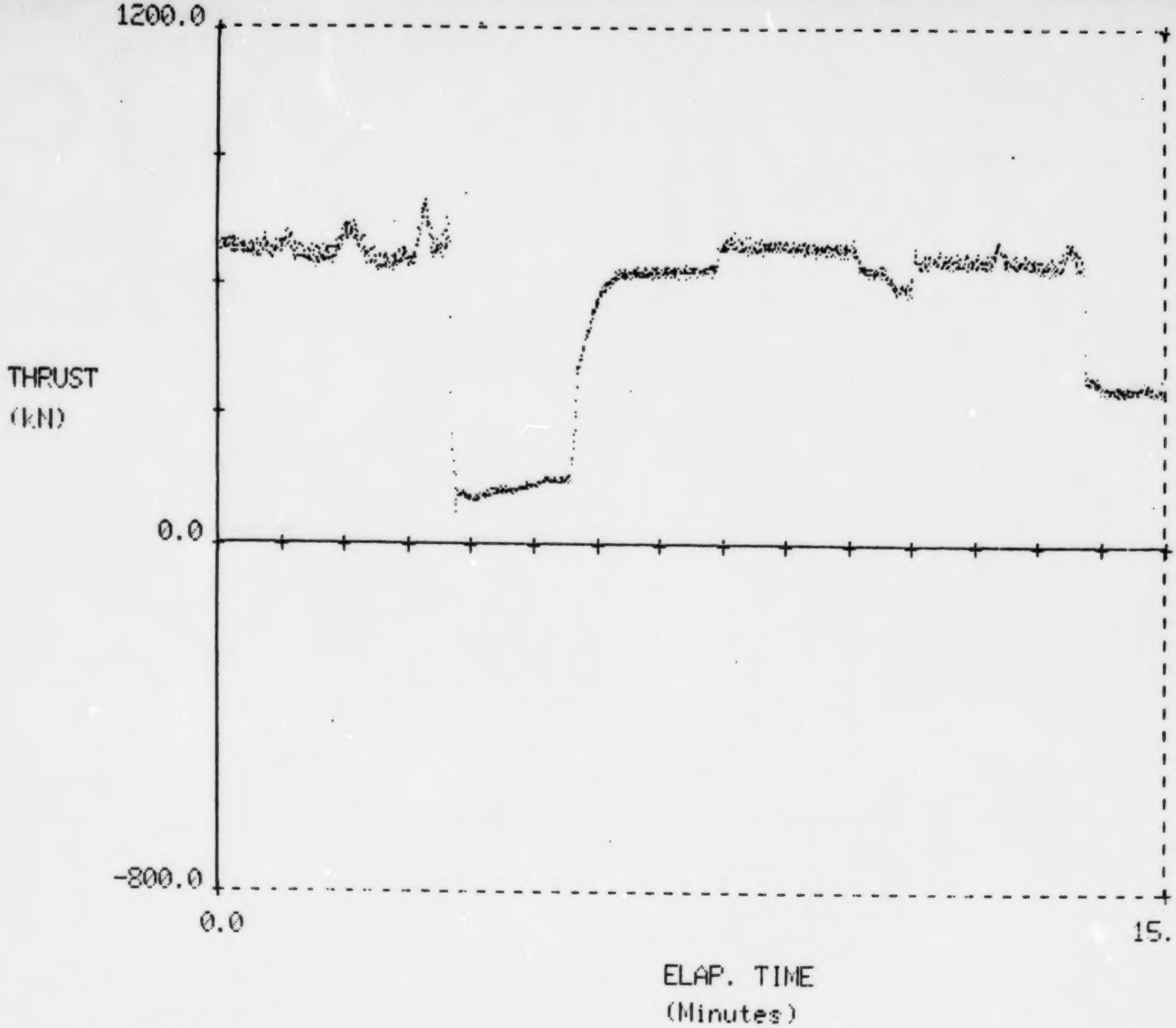
For continuous operation:- small bow angle at waterline; small spread angle complement; low kinetic friction; high bollard thrust.

For extraction:- low static friction; high backing thrust; large bow angle below design waterline.

APPENDIX B

SAMPLE PERFORMANCE PARAMETER

TIME HISTORY PLOTS



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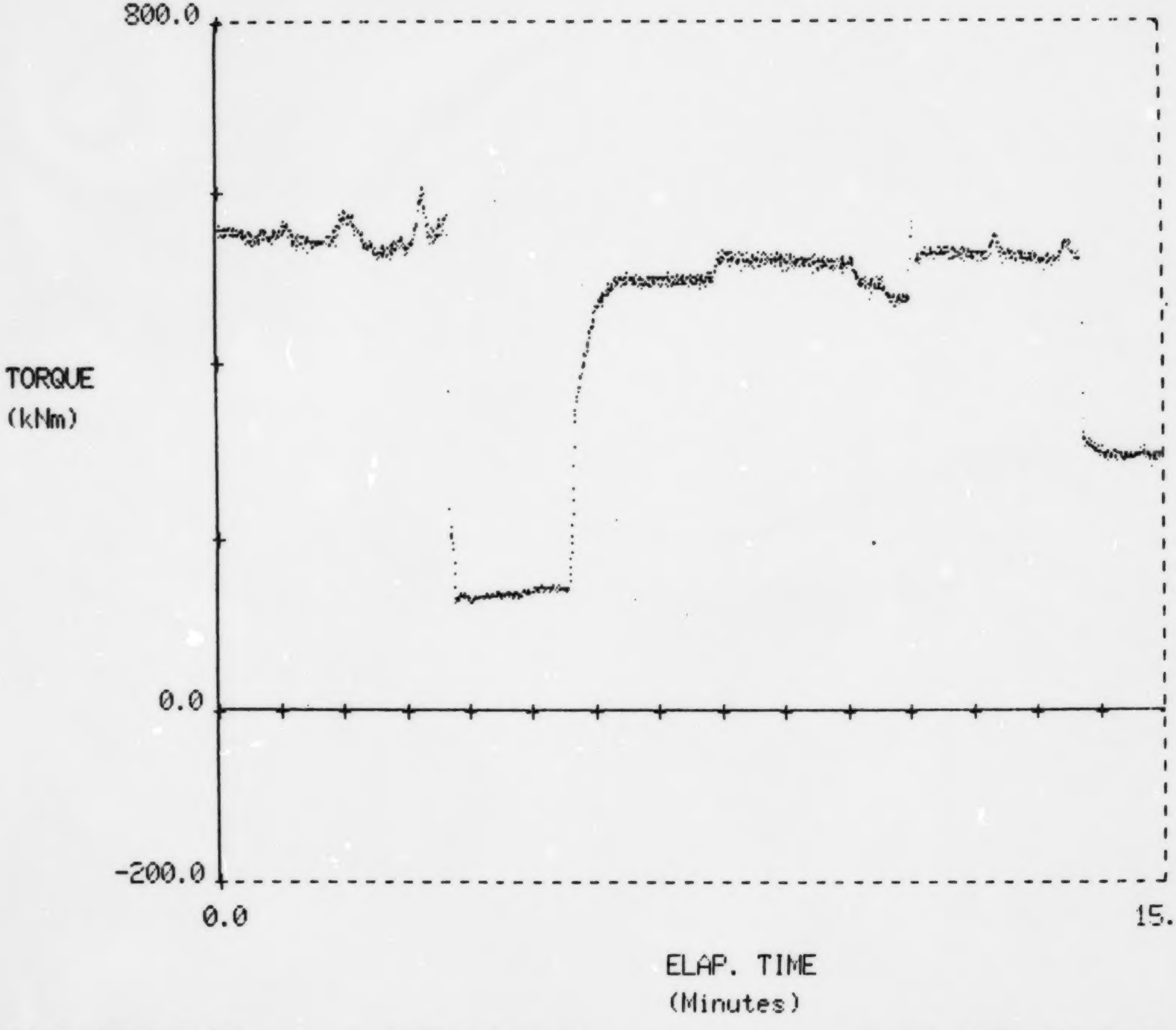

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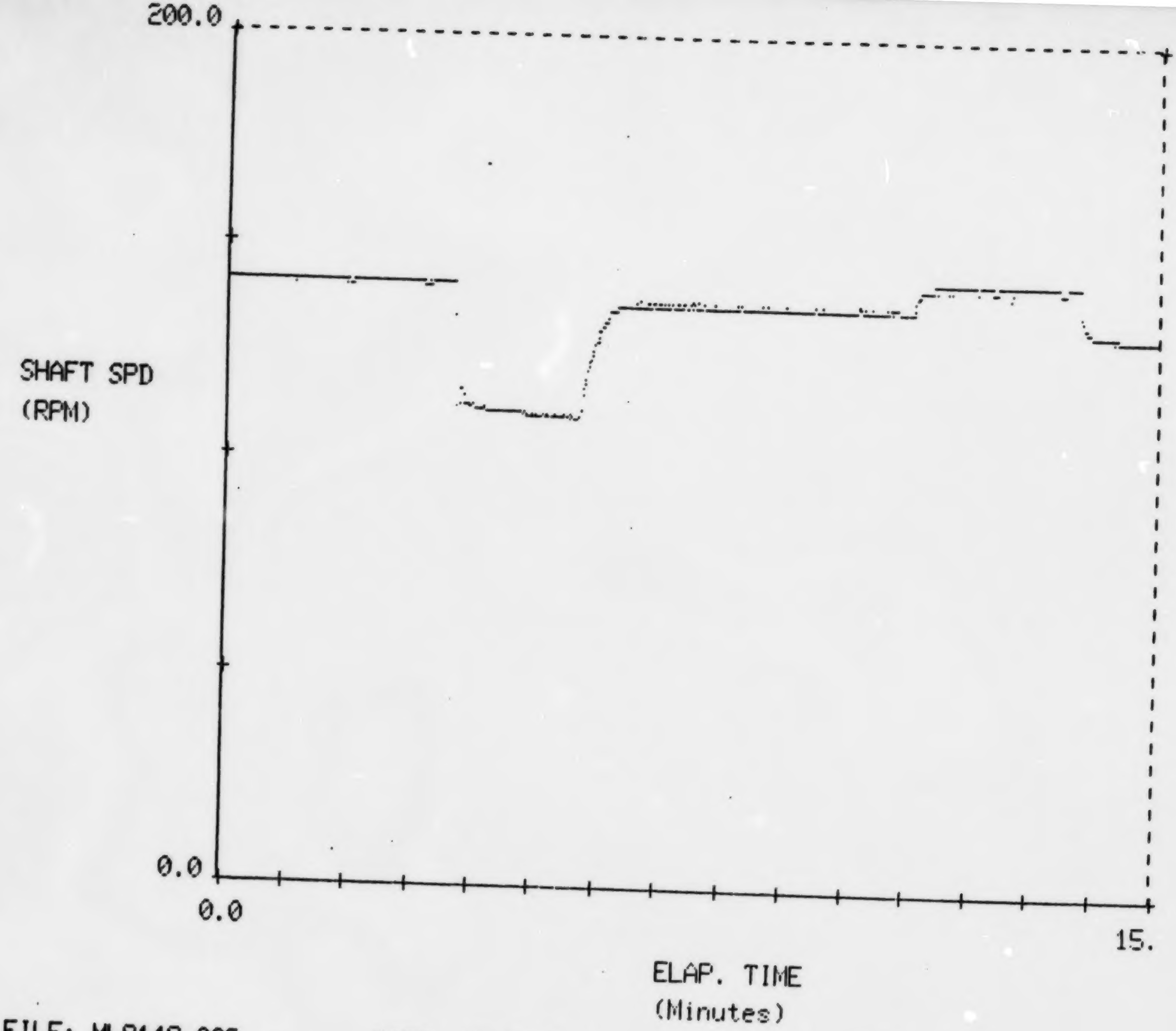

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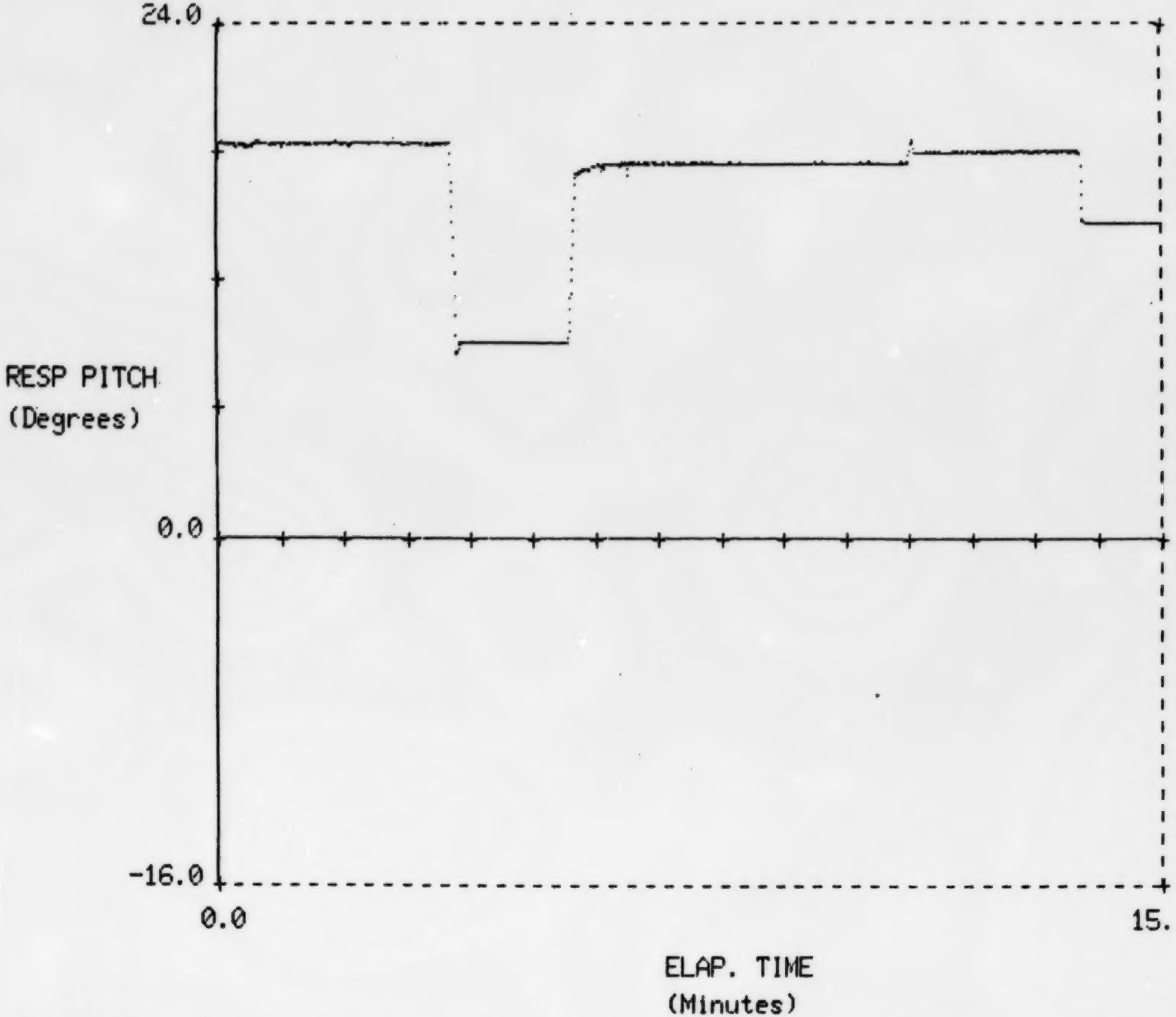

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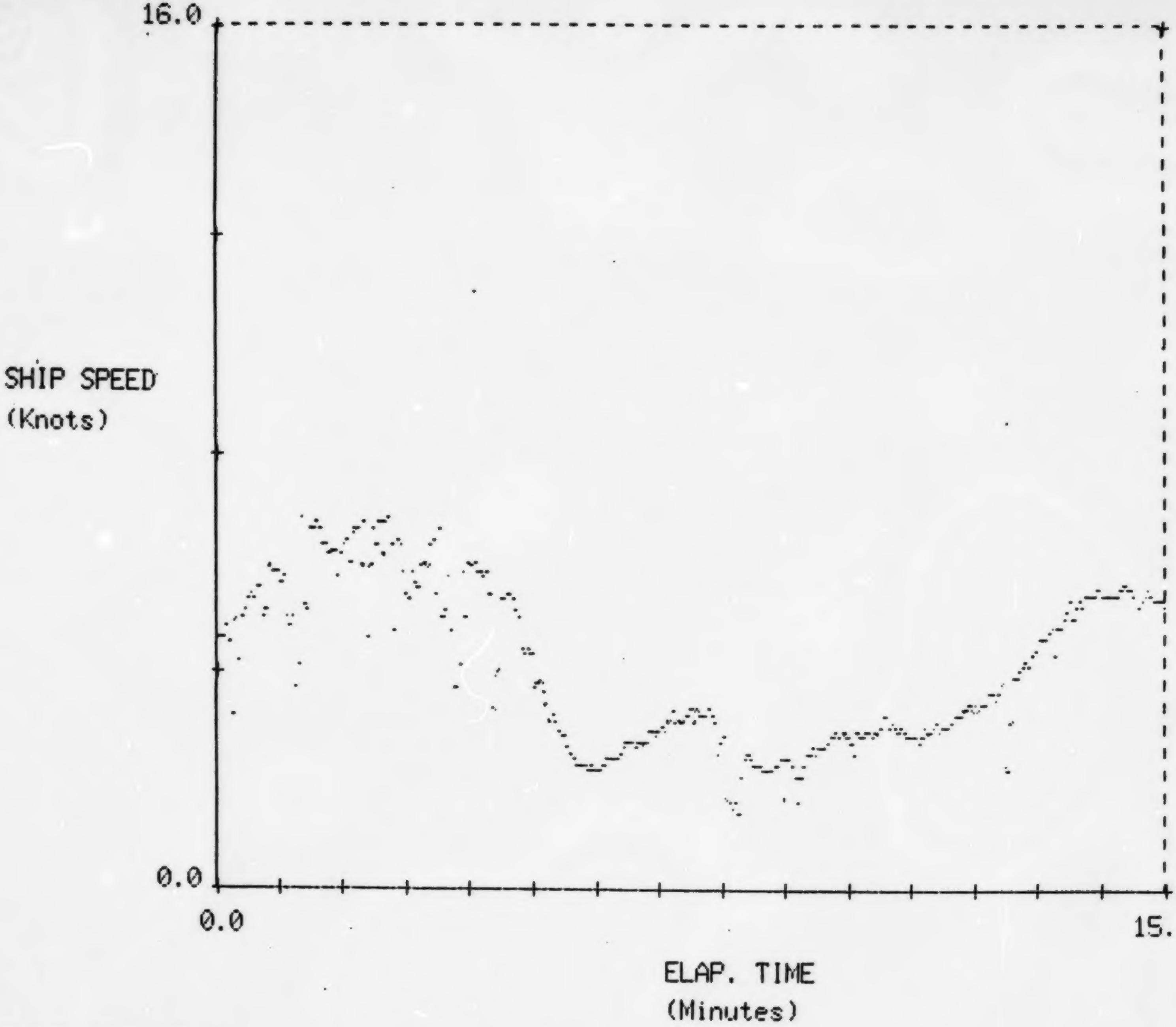

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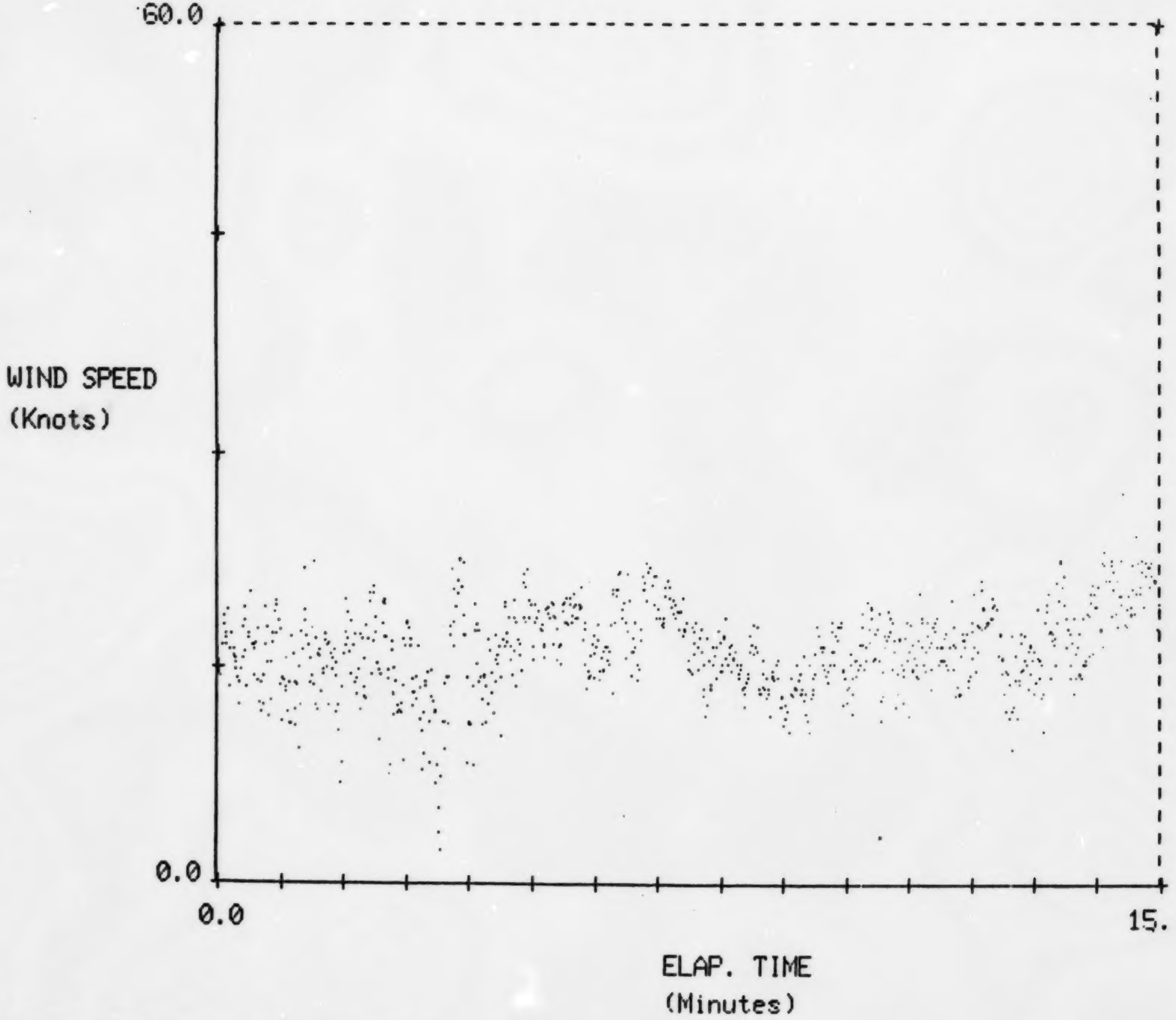

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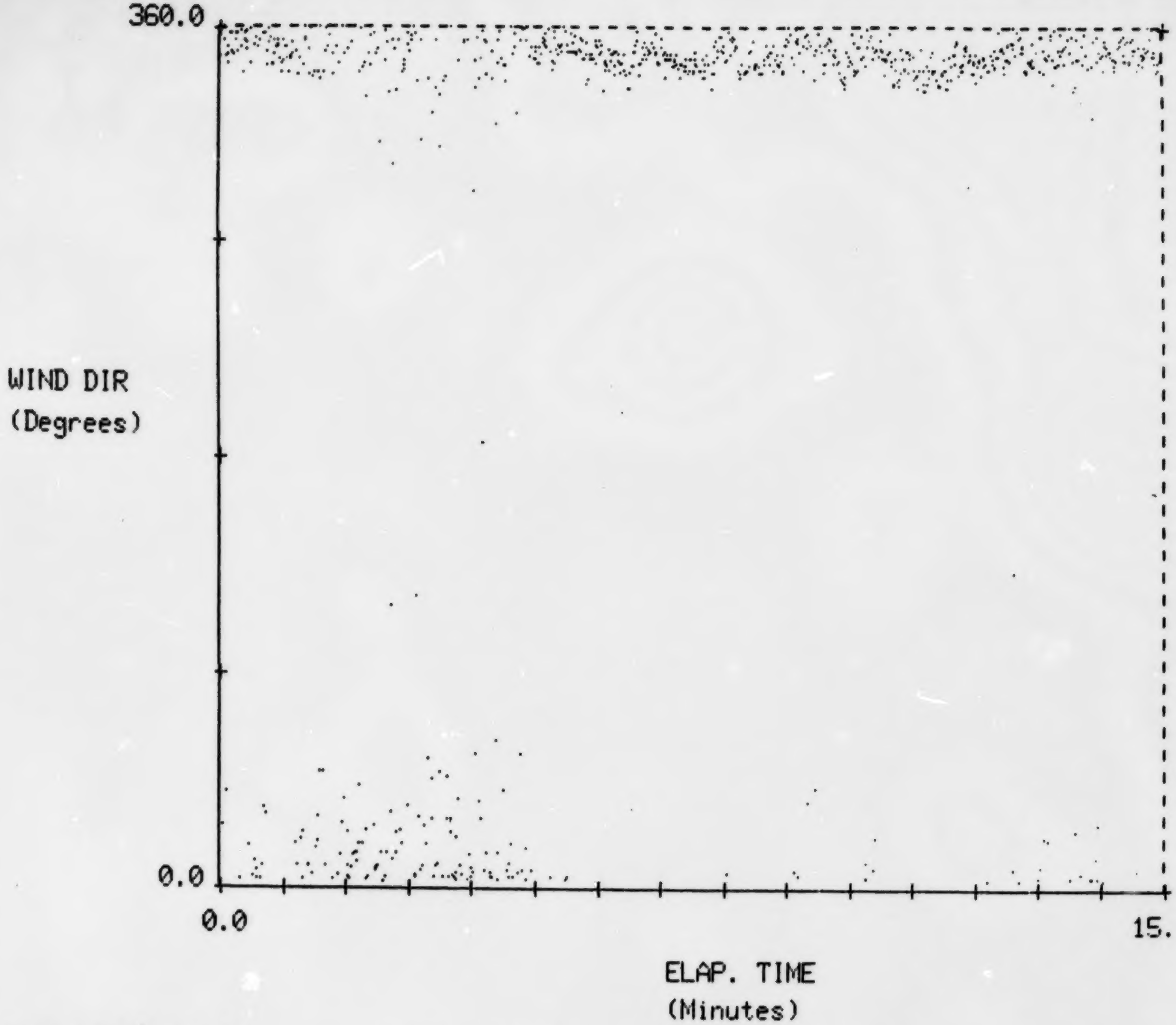

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APPENDIX C

MODEL AND TRIALS DATA ANALYSED AND CORRELATED IN RELATION TO THE PERFORMANCE EQUATION DEVELOPED BY S. T. MATHEWS OF COLDSTREAM MARITIME INC.

NOTE

The numerical values of thrust, propeller pitch and ice resistance used in this Appendix have been obtained by a slightly different procedure from that used in the main body of the report. However, the differences are not significant and do not affect the conclusions reached.

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NOMENCLATURE

1. ICE CHARACTERISTICS

σ	Tensile strength in flexure
ρ_i	Density, 934.2 Kg/m^3 (or N s^2/m^4), 0.9342 kN s^2/m^4
$\rho_i g$	Specific Weight, 9.162 kN/m^3
E	Modulous of Elasticity in flexure
e	$= (E/\sigma)(\rho_i/\sigma)$. Elasticity/strength coeff. $\left(\frac{s^2}{m^2}\right)$
σ_b	$= \sigma/\rho_i gb$ Non dimsl. strength rel. coeff.
e_b	$= egb$ Non dimsl. elasticity/strength rel. coeff.
h	Ice thickness, $h_b = h/b$
μ	Coefficient of friction, hull and ice

---

2. SECTION 1 AND SECTION 4

B	Ship beam $b = B/10$
V	Ship speed.
A	Form, ice breaking resistance const.
G, G_o	Energy transfer resistance const.
m	Longl. component factor tangl. forces
R_w	Ship resistance, clear water
R_i	Ice resistance
R_t	Total resistance in ice
C_i	Ice resistance coefficient
V_i	Ship speed in ice number
K_i	Ice resistance coefficient
$R_{wb} = \frac{Rw}{\rho_i g b^3}$	, Similarly for R_{ib}, R_{tb}
P_E	Ship hydrodynamic effective power
n	Propulsive Coefficient (clear water)
P_w	Delivered power (clear water)
P_t	Total delivered power in ice
s	Resistance / delivered power factor

$$P_{Eb} = \frac{P_E}{\rho g^{1\frac{1}{2}} b^{3\frac{1}{2}}}$$

Similarly for P_{wb} and P_{tb}

1.0 THE LEVEL ICE RESISTANCE EQUATION

In fluid mechanics, for plotting and presenting data, use is made of resistance coefficients of the type:

$$C = \frac{R}{\rho_w V^2 S} \quad \text{Where S is surface area}$$

which is a constant and gives a scaling law of comparison for geometrically similar bodies under dynamically similar conditions. (Rayleigh).

Conditions for dynamic similarity are: for viscous resistance, vl/ν = constant (Reynolds); surface effects, wavemaking resistance, $v/(gl)^{\frac{1}{2}}$ = constant (Froude); and for propellers, v/nd = constant, etc.

Constants of the above types have been usedin plotting, fairing and presenting the ship hydromechanics model data of Ref. C.3 which has been used as a standard for the M.V. Arctic clear water resistance, in these analyses.

There is a need to define the conditions for dynamic similarity for ice resistance and also the scaling law of comparison,as in Table 2.4.

This is especially the case for plotting, fairing and analyzing model and full scale trials data where it is difficult, in any test series, to hold constant the various independent parameters; h, σ, E, μ, and where there are, for the most part, too few data points for any one parameter.

An investigation into the ice resistance of icebreaking level ice has given the results described below.

The analysis considered the ice sheets breaking in various dynamic modes and included the ship Froude Number effects.

$$R_i = R_A + R_{Go} + R_\mu \tag{1}$$

R_i is the total ice resistance

R_A is the resistance to break ice

R_{Go} is the resistance associated with transferring energy to the broken ice, potential energy (submergence) and kinetic energy (motion of blocks).

R_μ is the resultant longitudinal component of the tangential forces due to friction between the ice blocks and the hull.

R_μ is considered as:

$R_\mu = m\mu \, R_{Go}$

Equation 1 becomes:

$$R_i = R_A + R_{Go} (1 + m\mu) \tag{2}$$

The analysis yielded:

$$R_A = A\sigma b h^{1\frac{1}{2}} g^{\frac{1}{2}} / (E/\rho_i)^{\frac{1}{4}} V^{\frac{1}{2}} \tag{3}$$

Where A is a constant for any particular hull form, and

$$R_{Go} = G_O \, \rho_i g^{\frac{1}{2}} \, b h^{1\frac{1}{2}} (E/\rho_i)^{\frac{1}{4}} V^{\frac{1}{2}} \tag{4}$$

Also where G is a constant for any particular hull form.

It should be noted that equations 3 and 4 are for the components of ice resistance. For total resistance in ice, including hydrodynamic resistance, when making model/ship comparisons, or comparing vessels of different sizes for the same form, the general scaling laws including the Froude condition must be met. These conditions are summarized in Table 4 of the Ice Committee report of Ref. 2, and in other publications.

Equations 3 and 4 show how the components of ice resistance vary with the independent parameters.

Both components are proportioned to $bh^{1\frac{1}{2}}$. As one would expect, the breaking component is proportional to σ.

The product $(E/\sigma)^{\frac{1}{4}}\ V^{\frac{1}{2}}$ is proportional to the width of the broken ice pieces.

R_A is inversely proportional to $(E/\sigma)^{\frac{1}{4}}\ V^{\frac{1}{2}}$ and R_{Go} is proportional to this product.

Equations 3 and 4 do not directly indicate a single non dimensional scaling law of comparison nor a single non-dimensional condition for dynamic similarity for total ice resistance.

For a constant value of $(E/\rho_i)^{\frac{1}{4}}\ V^{\frac{1}{2}}$ and a constant value of σ, however, R_i is proportional to $bh^{1\frac{1}{2}}$.

Substituting Equations 3 and 4 in 2 gives:

$$R_i = A\sigma bh^{1\frac{1}{2}}\ g^{\frac{1}{2}} \ /\ (E/\rho_i)^{\frac{1}{4}}\ V^{\frac{1}{2}} \ + \ (1 + m\mu)\ Go\ \rho_i g^{\frac{1}{2}}\ bh^{1\frac{1}{2}}\ (E/\rho_i)^{\frac{1}{4}}\ V^{\frac{1}{2}} \quad \text{---}\ (5)$$

Dividing through by $\sigma^{\frac{1}{2}}\ (\rho_i g)^{\frac{1}{2}}\ bh^{1\frac{1}{2}}$ gives

$$\frac{R_i}{\sigma^{\frac{1}{2}}\ (\rho_i g)^{\frac{1}{2}}\ bh^{1\frac{1}{2}}} = \frac{A}{((E/\sigma)\ (\rho_i/\sigma))^{\frac{1}{4}}\ V^{\frac{1}{2}}} + G\ ((E/\sigma)\ (\rho_i/\sigma))^{\frac{1}{4}}\ V^{\frac{1}{2}} \quad \text{-----}\ (6)$$

which gives:

$$C_i = \frac{A}{V_i^{\frac{1}{2}}} + GV_i^{\frac{1}{2}} \quad \text{--------------------------------------}\ (7)$$

Where $C_i = \dfrac{R_i}{\sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}}}$, $V_i = e^{\frac{1}{2}} V$

$e = (E/\sigma)(\rho_i/\sigma)$, $G = (1 + m\mu)\, Go$

From Equation 7:

$K_i = A + GV_i$ Linear Form ------------------ (8)

Where $K_i = C_i V_i^{\frac{1}{2}} = \dfrac{R_i\, e^{\frac{1}{4}}\, V^{\frac{1}{2}}}{\sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}}}$

Equations 6, 7 and 8 show that for a constant value of the non dimensional ship speed in ice coefficient V_i (or $e^{\frac{1}{2}} V$), resistance coefficients C_i and K_i are constants.

The ship speed in ice number V_i (or $e^{\frac{1}{2}} V$) being constant may be considered as a condition for dynamic similarity of total ice resistance*.

Under this condition, the total ice resistance* coefficient C_i then is constant and may be considered as giving the scaling law of comparison for total ice resistance:

For V_i = constant, total ice resistance* is proportional to the product $\sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}}$.

* Note: Total resistance in ice is equal to total ice resistance plus ship hydrodynamic resistance.

2.0 M.V. ARCTIC ICE RESISTANCE DATA

2.1 1974 Model Experiments

The model data are given in Table C.1A, having been extracted from Ref. C.1. Model clear water resistances (Rwm), allowing for the scale of 1/36, based on results in Ref. C.3 are subtracted from the measured total resistances to give the R_i values. In calculating coefficients C_i and V_i, E/σ has been taken as 2,000 since no value was given in the model report. The value of 2,000 was taken from Table 1 of the Ice Committee Report of Ref. C.2, as applying to the higher ice strengths in liquid nitrogen frozen ice. The non-dimensional data plots $K_i \sim V_i$ are shown in Fig. C.1A. The effect of friction is shown on Fig. C.1B and the method of calculating the effect of friction on resistance from the measured model data is described in Section 5 of this Appendix.

The model data can be represented by the equation:

$$K_i = 7 + 9.56\, V_i \,, \quad \text{for the effective } \mu = 0.178$$

and $$K_i = 7 + (1 + 13.8\mu)\, 2.77\, V_i \,, \quad \text{for all } \mu \text{ values.}$$

2.2 Lake Melville Trials 1981

The data have been lifted from the plots of Ref. C.4 and are given in Table C.2. The ice thicknesses are the averages for the bag numbers listed and the thicknesses are from Table 7.4 of Ref. C.5. The bag numbers corresponding to the runs have been taken from the charts of App. 3 of Ref. C.5 and correspond also to the times of day of Table 7.1 of Ref. C.5.

In Ref. C.5 stopwatch speeds were quoted for the runs and these speeds were used to obtain the calibration equation $V_s = 0.772 + 0.943\, V_c$ knots. Where V_s is the true speed and V_c is that from the computer tapes. The stopwatch speeds for four runs were omitted in obtaining this calibration since they were out of character with the other data and gave rise to wide scatter in the plotted results.

In Section 4 of the Main Report it is shown that there is good correlation between computer thrust and torque measurements and those of Ref. 3.

The corresponding model propeller pitch values were, however, on an average 3.5^{o} higher. These correlated values agree more closely with the manually recorded values of Ref. C.5.

The K_i V_i values for Lake Melville, bubbler off and on, are plotted in Figs. C.2 and C.3 respectively.

2.3 Baffin Island Trials 1980

The data points are given in Tables C.3A and C.3B. The ice thicknesses are from Tables A1 and 5.1 of Ref. 6. Stopwatch speeds were measured for most of the runs and these were used to obtain calibration equations for correcting the computer tape speeds.

There was a change in the stopwatch and tape speed relationship at Run 24.

The calibration equations were as follows:

Runs 1.1 to 23 inclusive, $V_s = 0.924 + 1.03\ V_c$ Knots
Runs 24 to 35.2 inclusive, $V_s = 0.088 + 1.02\ V_c$ Knots

The K_i V_i values are plotted in Figs. C4, C5 and C6. The results are separated into three groups as follows:

1. Bubbler Off, Strathcona Sound, Fig. C.4
2. Bubbler On, All areas, Fig. C.5
3. Bubbler Off, Navy Bd. In. and Lancaster Sd., Fig. C.6

One point, Run 1.4, plots high outside the diagram Fig. C.6.

2.4 Saguenay River Trials 1980

The data points are given in Table C.4a and the C_i, V_i, K_i coefficients in Table C.4b. The ice thickness values are from Ref. C.7.

No measurements of ice bending strength and Modulous of Elasticity were made.

An average ice temperature and an average salinity are quoted in Ref. C.7. These values and all the other statistical ice properties for the M.V. Arctic related ice tests were used to arrive at the ice strength and modulous figures quoted in Table 4b and used to calculate C_i, V_i and K_i.

$K_i \sim V_i$ is plotted in Fig. C.7.

3.0 REVIEW OF RESISTANCE ANALYSES AND CORRELATIONS

The constants in the resistance coefficient (C_i or K_i) equation, as developed in Section 1, calculated for the model in Section 5 and illustrated on the data plots of Figs. C1A to C.7, are summarized in Table C.5.

The model tests give the only data where friction coefficients were measured, and it is from the model data that the value of m was obtained. Similarly it is from the model data by successive iterations of regression analysis fits to K_i ($\mu_{av.}$) V_i, and $\frac{1 + m\mu}{1 + m\mu_{av.}} \sim \mu$ that the value of A was determined. An A value determined from any full scale data is susceptible to the data points scatter and the more likely errors in overall full scale instrumentation calibrations. In all cases the mean lines drawn through the data plots are such that the first moment of the differences from the mean line for each data set is zero.

Apart from the two model resistance values at $\mu = 0.03$, all the other model points are at friction values, $\mu = 0.132$ and 0.272, much higher than correlation values found for the trials data analyzed.

High friction is most often associated with heavy snow cover, and since this was low during the trials the results perhaps should not be unexpected.

The question arises as to whether more runs should not be made in model tests at lower friction values without the use of so many at high levels of artificial roughening.

The use of $\mu = 0.15$ as a predictor friction level in Ref. C.1 clearly, in the light of the trials data, leads to predicted resistances up to 2.4 times those possible in operation ($\mu = 0.02$), as was evident in the predictions from Ref. 1.

It should be emphasized that in Table 5 the μ values quoted are correlation values and when used with the value of $m = 13.8$ do give quite accurate representations of ice resistance.

The value of m is however, based on limited model data and any change in m would lead to different values of correlation μ without altering the resistance values.

The effect of speed (V_i) is such that, at the higher icebreaking speeds, the energy transfer term involving G becomes dominant.

For the Lake Melville Trials the effect of the bubbler on resistance varies from little or no effect at low speeds to increasing resistance up to 14% at higher speeds.

For the Saguenay River Trials there are only three runs with bubbler on and these would, on an average, indicate the same trend as for Lake Melville. The increase in resistance at the higher speeds in the Saguenay due to bubbler on being up to 6%.

If one compares the Baffin Island data, Strathcona Sound bubbler off, with all the Baffin Island data bubbler on, the Lake Melville-Saguenay trend is marginally reversed. The bubbler on data, Baffin Island gives rise to resistances up to 4% better than with bubbler off.

It was noticed by examining the basic data, T_p, R_i, V_s and h, for the bubbler off runs in Navy Board Inlet and Lancaster Sound, Baffin Island 1980 that, on an average, the resistance and thrust values were higher for given speeds and ice thicknesses than for those in Strathcona Sound.

The Navy Board Inlet and Lancaster Sound resistances were also high compared with the Lake Melville and Saguenay River values.

There was no indication of heavier snow coverage or any other reason for increased friction, in a conventional sense, in Navy Board Inlet or Lancaster Sound.

The equivalent or correlation friction in this area was three times the average at other locations.

Also here the data was scattered compared with the excellent uniformity of the values for the other trials locations.

The only explanation that can be advanced for this result is that there was a small degree of ice pressure present during the Navy Board Inlet and Lancaster Sound runs.

The results also indicate that the effects of pressure are offset with the bubbler system on.

Reinforcement to this explanation is given by the information in Ref. C.6, that there was rafting and a noticeable W-E current in the Lancaster Sound - Navy Board Inlet region.

4.0 TOTAL RESISTANCE AND POWER

4.1 Dimensional Equations

$$\text{Total resistance } R_T = R_i + R_w \qquad (1)$$

Where R_w is clear water resistance

From equation 7 of Section 1, substituting in 1:

$$R_T = \sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}} (A/V_i^{\frac{1}{2}} + GV_i^{\frac{1}{2}}) + Rw \qquad (2)$$

$$G = (1 + m\mu) Go \qquad (3)$$

In general the following relationships obtain:

Push available to break ice:

$$R_i = ((\rho_i^{\frac{1}{2}} b P_T)^{2/3} - (\rho_i^{\frac{1}{2}} b P_w)^{2/3}) s \qquad (4)$$

where P_w is the clear water power, S a function of V and pitch substituting in 1,

$$R_T = ((\rho_i^{\frac{1}{2}} b P_T)^{2/3} - (\rho_i^{\frac{1}{2}} b P_w)^{2/3}) s + R_w \qquad (5)$$

where P_E is effective power, η propulsive coefficient,

$$P_w = P_E/\eta \qquad (6)$$

From 4 and equation 7 of Section 1:

$$((\rho_i^{\frac{1}{2}} b P_T)^{2/3} - (\rho_i^{\frac{1}{2}} b P_w)) s = \sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}} (A/V_i^{\frac{1}{2}} + GV_i^{\frac{1}{2}}) \qquad (7)$$

7 is an equation giving total power for any ice thickness, strength and elasticity.

4.2 Empirical Formulae For M.V. Arctic

Equations 1 to 7 are general for any vessel. Following are simplified but accurate formulae for the M.V. Arctic based upon the information in Ref. C.3.

$$R_w = \rho_i g\, b^3\, (1.6 + 0.92\,(V_b - 1.06)^2)\; V_b^{\,2} \text{ ---------------- } (8)$$

$$V_b = V/(gb)^{\frac{1}{2}}$$

$$P_E = \rho_i g^{1\frac{1}{2}}\, b^{3\frac{1}{2}}\, (1.6 + 0.92\,(V_b - 1.06)^2)\; V_b^{\,3} \text{ ------------ } (9)$$

$$s = (2.04 - 0.0312\,(22.5 - p^{\circ}) - 0.513\, V_b^{\frac{1}{2}} \text{ --------------- } (10)$$

Where p° is propeller pitch in degrees.

$$\eta = 0.125\; (p^{\circ})^{\frac{1}{2}} \text{ --- } (11)$$

Formulae 8 to 11 may be substituted in equations 2, 4, 5 and 7 to calculate the various performance parameters.

Figures 8A to 8D were constructed using the above to illustrate the relationships between the various parameters and total resistance and power.

Clearly using these relationships many other types of diagram may be constructed.

4.3 Non-Dimensional Equations

For comparing forms of different sizes:

From Section 1:

$$C_i = \frac{R_i}{\sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}}} = (A/V_i^{\frac{1}{2}} + GV_i^{\frac{1}{2}}) \tag{12}$$

$$K_i = C_i V_i^{\frac{1}{2}} = \frac{R_i e^{\frac{1}{4}} V^{\frac{1}{2}}}{\sigma^{\frac{1}{2}} (\rho_i g)^{\frac{1}{2}} bh^{1\frac{1}{2}}} = A + GV_i \tag{13}$$

Define: $R_{ib} = \dfrac{R_i}{\rho_i g b^3}$, $R_{wb} = \dfrac{R_w}{\rho_i g b^3}$, $R_{TB} = \dfrac{R_T}{\rho_i g b^3}$

$$P_{Eb} = \frac{P_E}{\rho_i g^{1\frac{1}{2}} b^{3\frac{1}{2}}} , \quad P_{wb} = \frac{P_w}{\rho_i g^{1\frac{1}{2}} b^{3\frac{1}{2}}} , \quad P_{Tb} = \frac{P_T}{\rho_i g^{1\frac{1}{2}} b^{3\frac{1}{2}}}$$

Then: $$R_{ib} = \sigma_b^{\frac{1}{2}} h_b^{1\frac{1}{2}} (A / e_b^{\frac{1}{4}} V_b^{\frac{1}{2}} + Ge_b^{\frac{1}{4}} V_b^{\frac{1}{2}}) \tag{12A}$$

Where: $\sigma_b = \sigma/\rho_i gb$, $h_b = h/b$, $e_b = \dfrac{E/\sigma}{\sigma_b}$, $V_b = V/(gb)^{\frac{1}{2}}$

$$R_{Tb} = R_{ib} + R_{wb} \tag{1A}$$

$$R_{Tb} = \sigma_b^{\frac{1}{2}} h_b^{1\frac{1}{2}} (A / e_b^{\frac{1}{4}} V_b^{\frac{1}{2}} + Ge_b^{\frac{1}{4}} V_b^{\frac{1}{2}}) + R_{wb} \tag{2A}$$

Available push to break ice:

$$R_{ib} = (P_{Tb}^{2/3} - P_{wb}^{2/3})\, s \tag{4A}$$

$$R_{Tb} = (P_{Tb}^{2/3} - P_{wb}^{2/3})\, s + R_{wb} \tag{5A}$$

$$(P_{Tb}^{2/3} - P_{wb}^{2/3})\, s = \sigma_b^{\frac{1}{2}} h_b^{1\frac{1}{2}} (A / e_b^{\frac{1}{4}} V_b^{\frac{1}{2}} + Ge_b^{\frac{1}{4}} V_b^{\frac{1}{2}}) \tag{7A}$$

Empirical formulae 8 and 9 for the M.V. Arctic in non-dimensional form become:

$$R_{wb} = (1.6 + 0.92(V_b - 1.06)^2)\ V_b^2 \quad \text{----------------------} \quad (8A)$$

$$P_{Eb} = (1.6 + 0.92(V_b - 1.06)^2\ V_b^2 \quad \text{----------------------} \quad (9A)$$

Equations 3, 6, 10 and 11 are the same for dimensional and non-dimensional forms.

These non-dimensional equations were used to construct Figs. 9A to 9D showing the relationships between the various non-dimensional parameters and the resistance and power coefficients.

Such diagrams are useful for comparing results and making predictions for vessels of different sizes in ice of different thicknesses, strengths, elasticities and with different friction coefficients.

5.0 DERIVATION OF A, G, μ (EFFECTIVE AVERAGE), m, AND Go FROM THE 1974 MODEL DATA

From Section 1 $K_i = A + (1 + m\mu)\, G_o\, V_i$ --------------- (1)

A mean line, linear regression analysis equation, was first calculated for the $K_i\ (\mu) \sim V_i$ test data (column 4, Table C.1B) representing all the test μ values.

$\frac{1 + m\mu}{1 + m\mu_{av.}}$ was next calculated for each test point from the relationship

$$\frac{1 + m\mu}{1 + m\mu_{av.}} = \frac{K_i\ (\mu) - A}{K_i\ (\text{Equn.}) - A} \qquad \text{------------------------------ (2)}$$

A linear regression analysis of $\frac{1 + m\mu}{1 + m\mu_{av.}} \sim \mu$ then led to values of m and $\mu_{av.}$

The test points $K_i\ (\mu)$ were then corrected to values for $\mu = \mu_{av.}$ by the relationship:

$$K_i(\mu_{av.}) = (K_i(\mu) - A)\,.\,\frac{1 + m\mu_{av.}}{1 + m\mu} + A \qquad \text{------------------ (3)}$$

using the value of $\frac{1 + m\mu_{av.}}{1 + m\mu}$ appropriate to μ.

A new linear regression analysis equation was calculated for these corrected $K_i\ (\mu_{av.}) \sim V_i$ points.

This process was then repeated until the values of A, G, m and $\mu_{av.}$ suitably converged.

Columns 5, 6 and 7 of Table C.1B give the results of the third iteration.

The equation $K_i = 7 + 9.56\ V_i$ ---------------------------- (4)

is the result of a linear regression analysis on Column 6

$\dfrac{1 + m\mu}{1 + m\mu_{av.}}$ (Column 7) is plotted against μ in Fig. C1B. A linear regression analysis of Column 7 gives:

$$\frac{1 + m\mu}{1 + m\mu_{av.}} = 0.29 + 4\mu \quad \text{------------------------------------} \quad (5)$$

$$\text{For } \mu = 0 \qquad \frac{1}{1 + m\mu_{av.}} = 0.29 \quad \text{----------------------------} \quad (6)$$

Dividing Equation 5 by Equation 6 gives

$$1 + m\mu = 1 + 13.8\mu \quad \text{---------------------------------------} \quad (7)$$

When $\mu = \mu_{av.}$, from Equation 5

$$1 = 0.29 + 4\mu_{av.} \quad \text{---} \quad (8)$$

Equations 7 and 8 give $m = 13.8$, $\mu_{av.} = 0.178$

$$G_0 = \frac{G}{1 + m\mu_{av.}} = \frac{9.56}{1 + 13.8 \times 0.178} = 2.77$$

Hence $A = 7.0$, $m = 13.8$, $G_0 = 2.77$

Substituting in Eqn (1) for the M.V. Arctic

$K_i = 7 + (1 + 13.8\mu)\ 2.77\ V_i$ for all values of μ

as given in Sub-section 2.1 on Page C-5.

7.0 REFERENCES

C.1 Plate Ice Resistance Model Tests on a 28,000 DWT Arctic Class 2 Bulk Carrier, Arctec Canada Ltd., July 11, 1974.

C.2 15th International Towing Tank Conference Proceedings, the Hague 1978.

C.3 M. V. Arctic, Results of Resistance, Propulsion and Overload Experiments including Measured Duct Forces, National Research Council Canada LTR-SH-349, May 1983.

C.4 Trials Data Plots from Computer Tapes, Gas Tops Ltd., Ottawa 1983.

C.5 M. V. Arctic Experimental Voyage to Lake Melville, Labrador, in Feb./March 1981. 2 volumes, Melville Shipping Ltd. Transport Canada Report No. TP 3646E.

C.6 M. V. Arctic, Data Collection and Instrumentation, 1980 Season, Vol. 2 Baffin Is. Trials. Fenco Conslts. Ltd., and German & Milne Inc., Aug. 1981. Transport Canada Report No. TP 3202.

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TABLE C.1A

M.V. ARCTIC - 1/36 SCALE MODEL TESTS (1974)

E/σ ASSUMED AS 2,000

ARCTEC REPORT JULY 11, 1974

TEST	μ	h m	σ kPa	V m/s	R_{TM} kN	R_{wm} kN	R_i kN	C_i TEST μ	V_i	K_i TEST μ	K_i $\mu = 0.178$
2-1	0.272	1.004	374.2	4.200	5737	237	5500	40.84	9.385	125.1	92.86
3-1	0.272	0.522	469.5	4.182	2132	234	1898	33.57	8.343	96.96	72.40
3-2	0.272	0.4752	469.5	1.014	1373	19	1354	27.57	2.023	39.21	30.42
4-2	0.272	1.454	1585	2.064	11383	77	11306	23.41	2.241	35.04	27.39
4-4	0.272	1.318	1585	0.9540	9466	16	9450	22.67	1.036	23.07	18.68
5-1	0.272	0.6048	907.3	3.150	2402	149	2253	22.98	4.520	48.86	37.43
6-1	0.132	3.138	868.5	3.138	2626	146	2480	26.57	4.603	57.00	68.24
6-2	0.132	0.9660	868.5	0.9660	1675	17	1658	18.60	1.417	22.14	25.54
7-1	0.132	5.694	737.9	5.694	2118	412	1706	30.07	9.061	90.51	109.3
7-2	0.132	2.064	737.9	2.064	1158	77	1081	23.08	3.284	41.83	49.66
8-1	0.03	3.138	1112	3.138	1281	146	1135	9.124	4.068	18.40	34.86
8-2	0.03	0.9900	1112	0.9900	677	18	659	5.862	1.283	6.64	6.12

TABLE C.1B

M.V. ARCTIC - 1/36 SCALE MODEL TESTS (1974)

ANALYSIS FOR EFFECTS OF FRICTION

TABLE 1A

(1) TEST	(2) μ	(3) V_i	(4) K_i TEST μ	(5) K_i $\mu_{av.}$	(6) K_i Eq. $\mu_{av.}$	(7) $\frac{1 + m\mu}{1 + m\mu_{av.}}$
2-1	0.272	9.385	125.1	92.86	96.72	1.316
3-1	0.272	8.343	96.96	72.40	86.76	1.128
3-2	0.272	2.023	39.21	30.42	26.34	1.665
4-2	0.272	2.241	35.04	27.39	28.42	1.309
4-4	0.272	1.036	23.07	18.68	16.90	1.623
5-1	0.272	4.520	48.86	37.43	50.21	0.969
6-1	0.132	4.603	57.00	68.24	51.00	1.136
6-2	0.132	1.417	22.14	25.54	20.55	1.118
7-1	0.132	9.061	90.51	109.3	93.62	0.964
7-2	0.132	3.284	41.83	49.66	38.4	1.109
8-1	0.03	4.068	18.40	34.86	45.89	0.293
8-2	0.03	1.283	6.64	6.12	19.27	-0.029

EQUATION $K_i = 7 + 9.56 V_i$

TABLE C.2

1981 LAKE MELVILLE LEVEL ICE TRIALS DATA

NOTES:
1. Times from Table 7.1 Bag Nos. Charts App. 3
2. Table A 7.4
3. Adjusted by Equation from stopwatch calibration
4. 3.5 degrees added to tape-equation values.
5. By propeller thrust identity calculation.

REPORT TP 3646 E

				1	2		3								
FILE MM81	DATE 1981	RUN	BUBBLER	BAG #	h m	Vc m/s	Vs m/s	Tp kN	Q kN.m	Ns RPM	P DEG	Ri	Ci	Vi	Ki
09.001	27/2	1a		1-2	.640	.5402	.9067	360	248	123	14.7	528.2	9.124	5.002	20.41
		1b		1-2	.640	.7974	1.149	425	302	128	15.7	610.7	10.55	6.339	26.56
		2		2-3	.590	1.157	1.489	505	360	132	16.4	705.9	13.78	8.213	39.48
		3a		3-4	.640	1.878	2.168	620	435	138	17.4	814.9	14.08	11.96	48.68
		3b		3-4	.640	2.444	2.702	730	550	140	18.9	928.7	16.04	14.90	61.93
		4		4	.700	2.984	3.211	840	653	144	20.0	1035.1	15.63	17.71	65.79
		5	on	4-5	.635	1.080	1.416	600	445	136	17.4	858.6	15.01	7.811	41.95
		6	on	5-6	.670	1.749	2.047	735	550	140	18.9	1004.0	16.19	11.29	54.41
		7	on	6-7	.767	.2006	2.289	935	715	144	20.6	1285.0	16.92	12.63	60.12
08.001		8	on	9	.827	.8231	1.174	610	485	135	17.4	899.8	10.58	6.47	26.92
		9	on	10-11	.715	1.646	1.950	750	560	140	18.9	1037.4	15.18	10.76	49.77
		10	On	11-12	.803	1.903	2.192	910	720	144	20.4	1258.3	15.47	12.09	53.79
		11		12-13	.757	.3601	.7369	630	480	136	17.3	974.7	13.09	4.065	26.39
		12		13-14	.860	.7974	1.149	765	570	140	18.9	1148.0	12.73	6.339	32.06
05.001	1/3	13	On	18	.930	.4116	.7854	780	560	140	18.9	1213.9	11.97	4.332	24.92
		14	On	19-20	.900	1.132	1.465	930	712	144	20.6	1375.9	14.25	8.079	40.52
		15		21	1.67	1.029	1.368	945	725	14.4	20.6	1412.3	11.33	7.543	31.13
		16		22	.917	.6688	1.028	875	630	14.25	19.5	1338.5	13.48	5.671	32.11
		17		23	.860	1.749	2.047	940	710	144	20.6	1320.6	14.65	11.29	49.21
04.011		18	On	24	.795	1.595	1.901	960	706	144	20.5	1369.6	17.09	10.49	55.35

TABLE C.3A

1980 BAFFIN ISLAND LEVEL ICE TRIALS DATA

				TP 3202				†				††			
FILE MB80	DATE 1980	RUN	BUBBLER	h m	AREA	ICE TESTS	V_c m/s	V_s m/s	Tp kN	Q kN.m	p^o	R_i	C_i	V_i	K_i
01.001	26/10	1.1		0.183		σ = 342 KPa E = 1094 MPa E/σ = 3200	1.235	1.748	170	115	7.3	181.3	18.10	5.167	41.13
01.002		1.2		0.192			2.932	3.496	310	240	12.4	249.7	23.19	10.33	74.55
02.002		1.4		0.186			6.019	6.675	700	665	20.8	373.7	36.41	19.73	161.7
03.002		3.3		0.195			3.344	3.920	325	235	12.7	232.3	21.08	11.59	71.76
		3.4		0.224			1.518	2.039	175	107	7.8	170.4	12.56	6.027	30.84
05.004	27/10	6.1		0.312			0.617	1.112	180	110	8.0	233.7	10.48	3.287	19.00
05.005		6.2		0.325			2.315	2.860	335	242	12.6	336.7	14.20	8.454	41.29
05.006		6.3		0.307			4.296	4.901	615	485	16.4	512.3	23.54	14.49	89.58
05.010		9.3	ON	0.295		SITE 3 MID PT. BETWEEN RUNS	2.881	3.443	320	212	12.8	268.5	13.09	10.18	41.77
05.011		9.4	ON	0.293			1.029	1.536	170	90	7.85	195.1	9.615	4.540	20.49
05.012		10.1		0.300			0.823	1.324	171	96	7.9	208.9	9.933	3.914	19.65
05.016		10.3		0.300			3.704	4.291	460	336	14.4	374.4	17.80	12.68	63.41
05.014		10.4		0.318*			5.196	6.047*	800	690	20.8	610.9	26.68	17.88	112.8
08.004	28/10	15.0		0.318*			5.623	6.047*	800	690	20.8	610.9	27.17	20.35	122.6
10.003	29/10	22.0	ON	0.435	LANC. SOUND	σ = 330 kPa E = 132 MPa E/σ = 4000	1.286	1.720*	270	192	11.8	331.9	9.203	5.788	22.14
10.003		23.0	ON	0.435			1.132	1.720*	270	192	11.8	331.9	9.203	5.788	22.14
10.004		24.0		0.350			3.663	3.784	397	300	15.0	346.3	13.31	12.73	47.48
11.005		27.0		0.357			1.440	1.516	320	210	11.2	415.2	15.49	3.783	34.98

* Average of 2 runs

† Speed adjusted by equation stopwatch calibration

†† Corresponding to V_s and Tp Identity (Eqn.)

TABLE C.3B

1980 BAFFIN ISLAND LEVEL ICE TRIALS DATA

CONTINUED FROM TABLE 3A

				TP 3202			†					††			
FILE MB80	DATE 1980	RUN	BUBBLER	h m	AREA	ICE TESTS	V_c m/s	V_s m/s	T_p kN	Q kN.m	p^o	R_i	C_i	V_i	K_i
17.001	31/10	30.1		0.335	STRATHCONA SOUND	σ = 411 kPa E = 1403 MPa E/σ = 3415	1.183	1.253	183	128	10.4	234.5	8.619	3.491	16.10
17.002		30.2	ON	0.335			1.286	1.358	182	125	10.4	227.0	8.344	3.783	16.23
17.002		31.1		0.330			3.292	3.512*	360	280	14.0	318.9	11.99	9.784	37.51
17.002		31.2		0.330			3.498	3.512*	360	280	14.0	318.9	11.99	9.784	37.51
17.002		32.1	ON	0.345			4.116	4.247	450	375	16.4	375.6	13.21	11.83	45.45
17.002		32.2		0.345			4.013	4.142	450	373	16.4	386.2	13.59	11.54	46.15
18.001		33.1	ON	0.330			6.276	6.453	770	705	20.8	502.6	18.90	17.98	80.13
18.001		33.2		0.330			6.328	6.505	775	707	20.8	498.3	18.74	18.12	79.76
18.001		34.1	ON	0.330			4.424	4.562	485	378	16.2	385.8	14.51	12.71	51.71
18.001		34.2		0.330			4.373	4.509	497	380	16.2	407.0	15.30	12.56	54.24
18.001		35.1	ON	0.315			2.006	2.093	240	160	8.3	255.2	10.29	5.831	24.85
18.001		35.2		0.315			1.929	2.015	240	160	8.3	260.7	10.51	5.614	24.91

* Average of 2 runs

† Speed adjusted by equation stopwatch calibration

†† Corresponding to V_s and Tp Identity (Eqn.)

TABLE C.4A

1980 SAGUENAY RIVER LEVEL ICE TRIALS DATA

Manually Recorded TP 2627						From Tape Recordings							Derived	
Run	Date 1980	h m	Snow CVR cm	N RPM	BUBBLER	File MS80	Tp kN	Q kN.m	Vs Kn	p +++	Wind Speed Kn	Wind Dirn. Deg.	p Deg.	Rf* kN
1-DS	20/1	0.4445	2.5			01.001	100	70		0.4	16	240		
1-S(¼)				114.4			160	100	1.1	1.8	"	"	7.3	230
1-½				127.1			280	180	1.8	3.2	"	"	10.05	415
1-3/4				127.1			360	275	4.0	4.4	"	"	13.7	480
1-F				133.5			595	525	7.6	5.8	"	"	19.7	660
2-¼	20/1	0.4445	2.5	127.1		01.001	160	100		1.8	23	240		
2-½				127.1			280	180	1.59+	3.0	"	"	10.1	420
2-3/4				127.1			360	280	3.50+	4.2	"	"	13.7	505
2-F				133.5		02.001	580	525	7.1	5.8	17	240	19.7	660
4-S(¼)	21/1	0.4842	3.75	127.1		06.001	140	107		1.2	29	90		
4-½				127.1			175	120	1.35	1.6	"	"	7.0	270
4-3/4				127.1			260	167		3.0	"	"		
4-X				127.1			340	260	2.0	4.0	"	"	12.7	540
4-F				133.5			570	525	6.07	5.7	"	"	19.2	730
6-S	21/1	0.5017	3.75	114.4	ON	07.001	100	75		0.4	18	80		
6-½				114.4	ON		160	110		1.8	"	"		
6-3/4				127.1	ON		280	187	1.21+	2.9	"	"	10.1	440
6-X				127.1	ON		380	305	2.7	4.4	22	85	14.7	585
6-F				133.5	ON		640	540	5.9	5.8	30	80	20.7	815
7-½	21/1	0.3715‡	3.75	114.4		08.002	160	105	0.5	1.8	18	30	5.7	260
7-3/4				130.3			260	165	0.8	2.8	"	"	8.2	390
7-X				127.1			340	255	2.5	4.2	"	"	12.6	515
7-F				139.8			580	530	8.6	5.8	22	30	18.8	575

‡ mean of
0.4318
and
0.3112

+ off tapes
Report TP 2627

+++ Supposed to be degrees using tape conversion eqn. supplied.

* Thrust and torque identity

TABLE C.4B

1980 SAGUENAY RIVER LEVEL ICE TRIALS DATA

Continued From Table 4A

*
σ = 450 K Pd
E = 2250 M Pd
E/σ = 5000

* Values based on measured statistics for other trials, salinity and temperature

			1				
RUN	h m	V m/s	Ri KN	Ci	Vi	Ki	BUBBLER
1-DS	0.4445						
1-S(¼)		0.5659	230	5.287	1.823	7.139	
1-½		0.9260	415	9.540	2.983	16.48	
1-3/4		2.058	480	11.03	6.630	28.41	
1-F		3.910	660	15.17	12.60	53.85	
2-¼	0.4445						
2-½		0.818	420	9.655	2.635	15.67	
2-3/4		1.801	505	11.61	5.801	27.96	
2-F		3.653	660	15.17	11.77	52.05	
4-S(¼)	0.4842						
4-½		0.6945	270	5.459	2.238	8.166	
4-3/4							
4-X		1.029	540	10.92	3.315	19.88	
4-F		3.123	730	14.76	10.06	46.82	
6-S	0.5017						
6-½							
6-3/4		0.6225	440	8.435	2.006	11.95	ON
6-X		1.389	585	11.22	4.475	23.73	ON
6-F		3.035	815	15.63	9.779	48.86	ON
7-½	.3715	0.2572	260	7.823	0.8286	7.121	
7-3/4		0.4116	390	11.73	1.326	13.51	
7-X		1.286	515	15.50	4.143	31.54	
7-F		4.424	575	17.30	14.25	65.32	

TABLE C.5

EQUATION CONSTANTS

AND

MODEL CORRELATION FRICTION COEFFICIENTS

$$C_i = A/V_i^{\frac{1}{2}} + GV_i^{\frac{1}{2}}$$

$K_i = A + GV_i$ -- Linear in V_i

where $G = (1+m\mu)\,Go$ and $K_i = C_i V_i^{\frac{1}{2}}$

FOR MV ARCTIC

$A = 7^{\dagger}$ $m=13.8^{\dagger}$ $G_o = 2.77^{\dagger}$

	EXPERIMENTS	G	$1 + m\mu$	CORRELATION μ
BUBBLER OFF	1974 Model Tests	$9.56^{\dagger}$	$3.456^{\dagger}$	$0.178^{\dagger}$
	1981 Lake Melville	3.60	1.302	0.022
	1980 Baffin Is. Strathcona SD.	3.49	1.260	0.019
	1980 Baffin Is. NBI, Lanc. SD.	5.25	1.898	0.065
	1980 Saguenay River	3.68	1.330	0.024
BUBBLER ON	1981 Lake Melville	4.11	1.486	0.035
	1980 Baffin Is. all runs	3.34	1.208	0.015
	1980 Saguenay River 3 runs	3.91	1.413	0.030

† ANALYSIS OF MODEL DATA INCLUDING FRICTION MEASUREMENTS

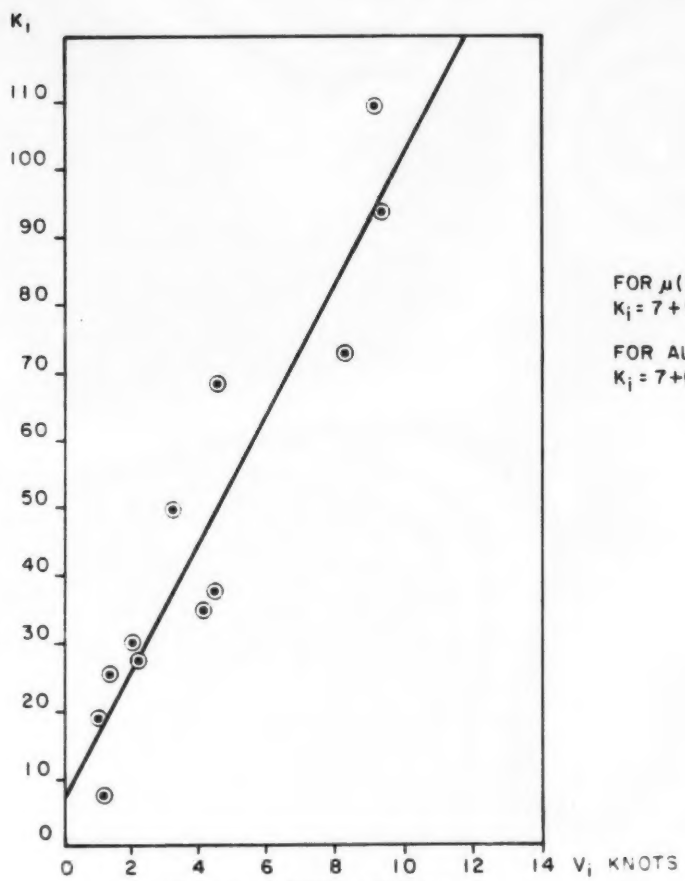

FIG. 1A MODEL (1974)

FOR $\mu(AV) = 0.178$
$K_i = 7 + 9.56 V_i$

FOR ALL μ
$K_i = 7 + (1 + 13.8\mu)\, 2.77 V_i$

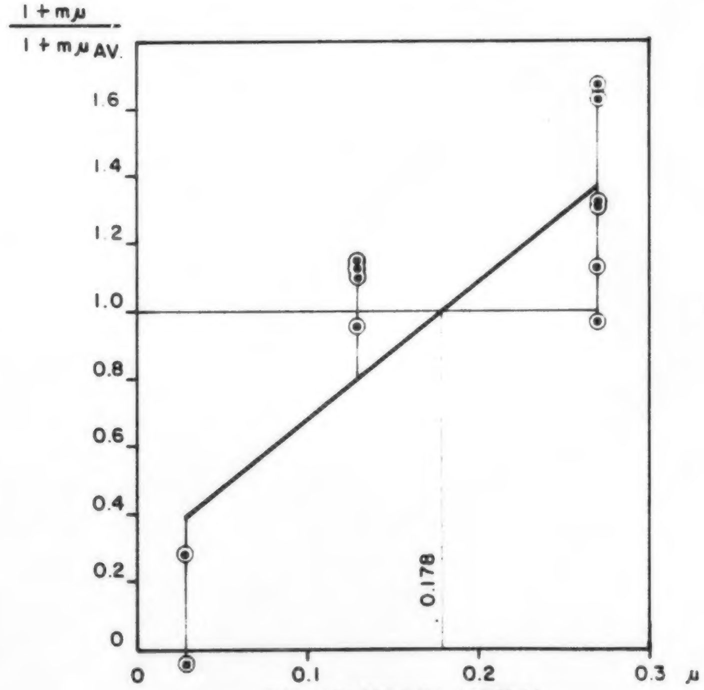

FIG. 1B MODEL (1974)

$$0.29 + 4\mu = \frac{1 + m\mu}{1 + m\mu_{AV.}}$$

M.V. ARCTIC LEVEL ICE DATA PLOTS, K_i V_i

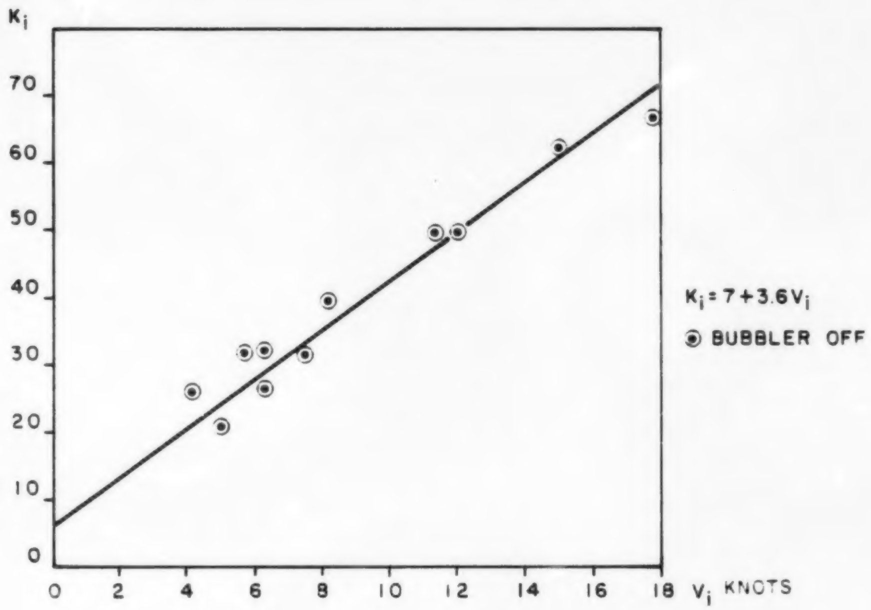

FIG. 2 LAKE MELVILLE

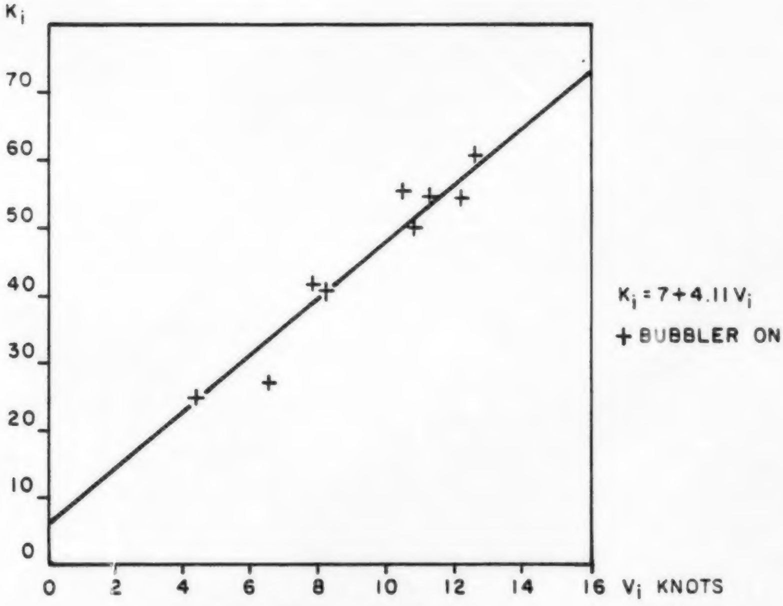

FIG. 3 LAKE MELVILLE

M.V. ARCTIC LEVEL ICE DATA PLOTS, K_i V_i

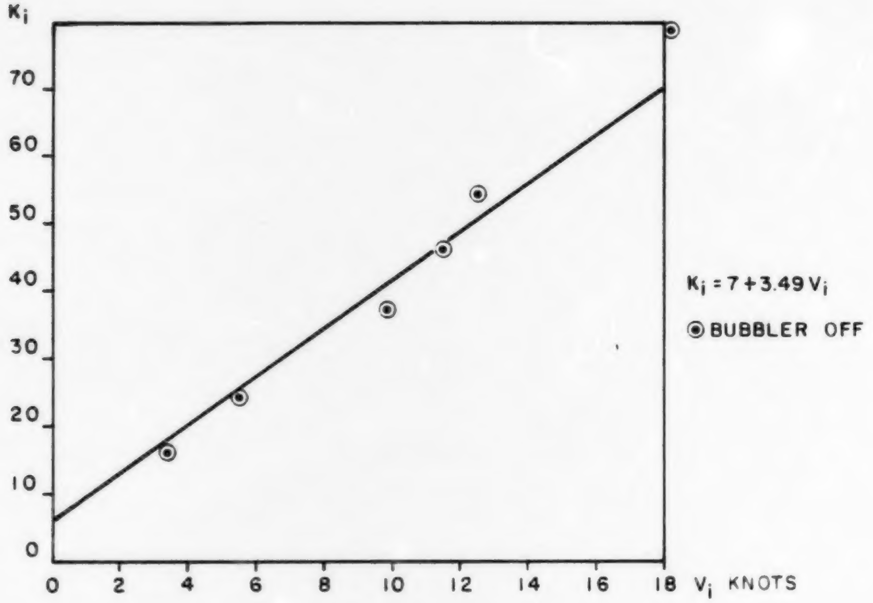

FIG. 4 BAFFIN ISLAND
STRATHCONA SOUND

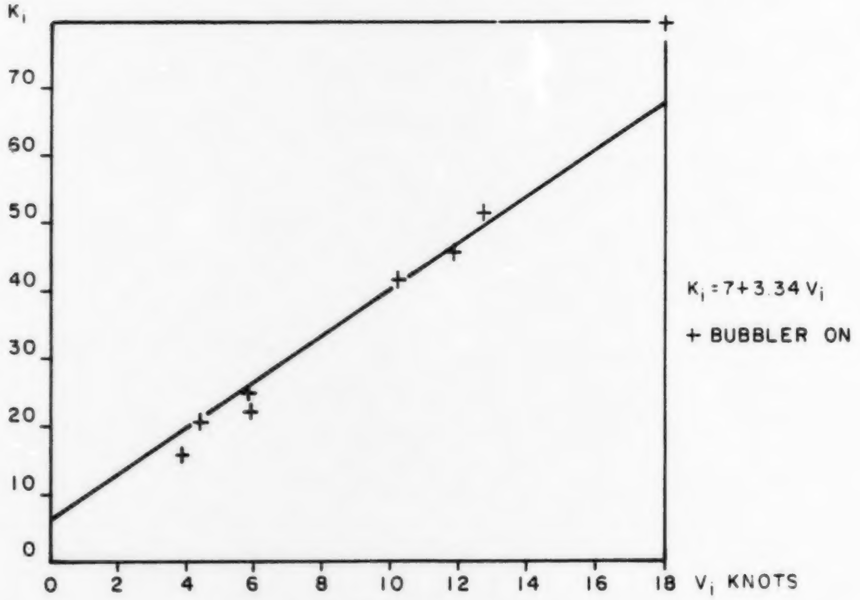

FIG. 5 BAFFIN ISLAND
STRATHCONA SOUND, NAVY BOARD IN.,
LANCASTER SOUND

M.V. ARCTIC LEVEL ICE DATA PLOTS, K_i V_i

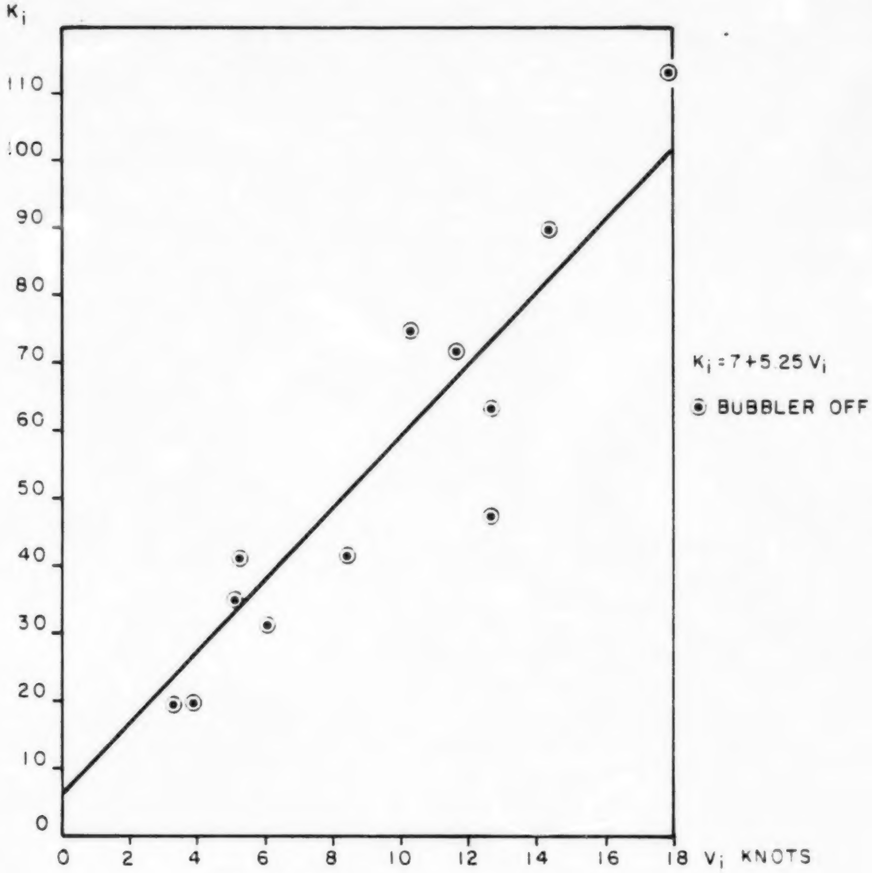

FIG. 6 BAFFIN ISLAND
NAVY BOARD IN., LANCASTER SOUND

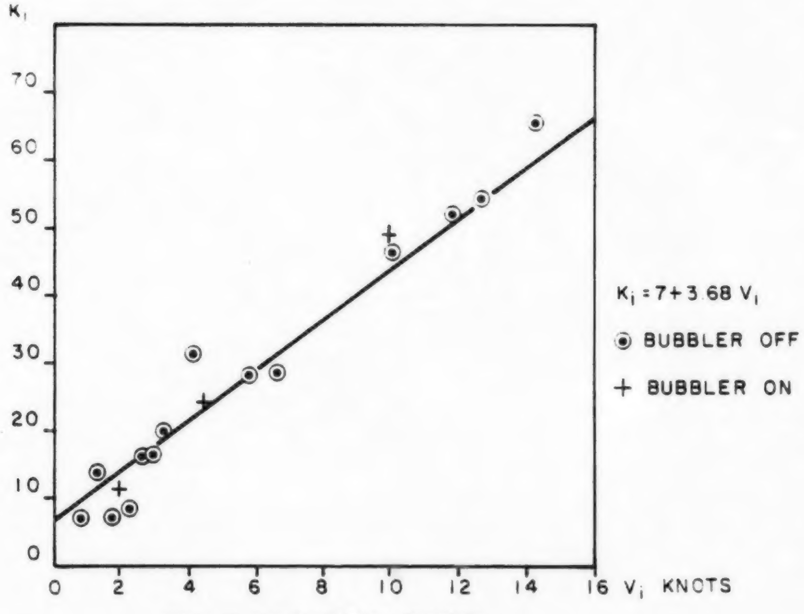

FIG. 7 SAGUENAY RIVER

M.V. ARCTIC LEVEL ICE DATA PLOTS, K_i V_i

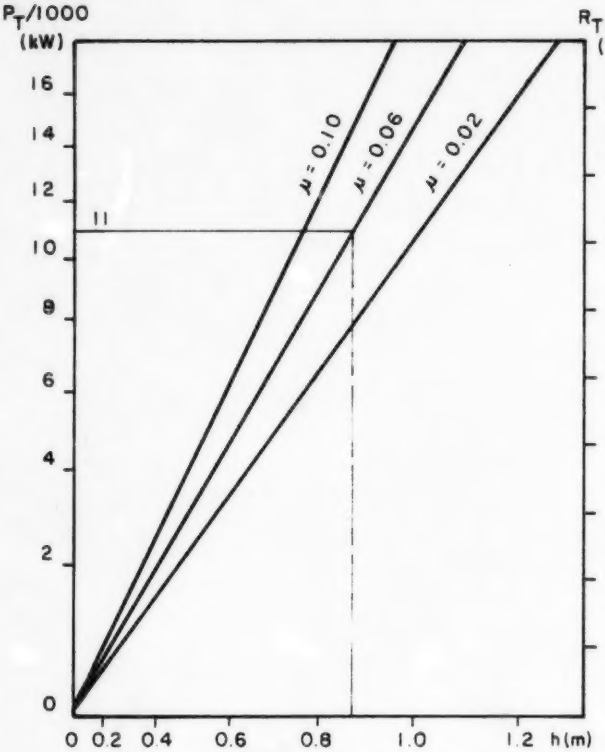

FIG. 8A V = 1 m/s

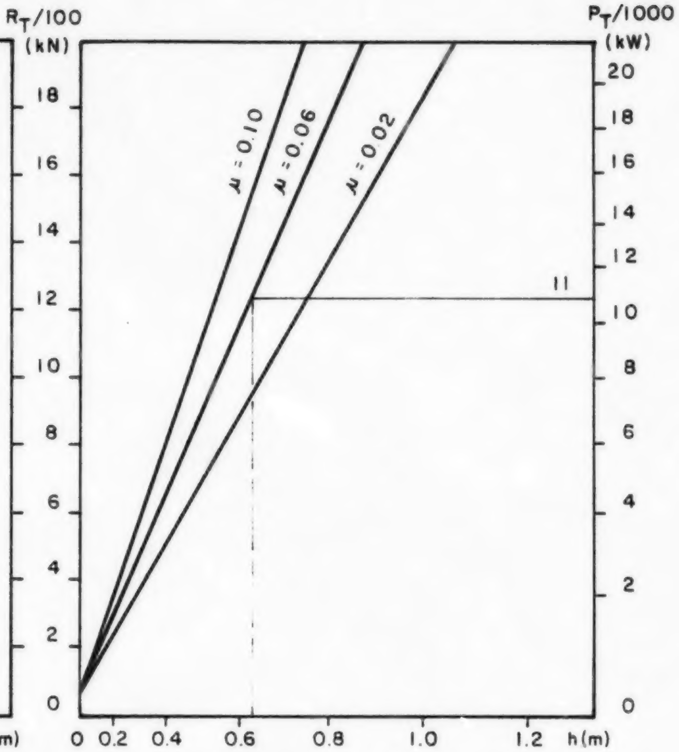

FIG. 8B V = 3m/s

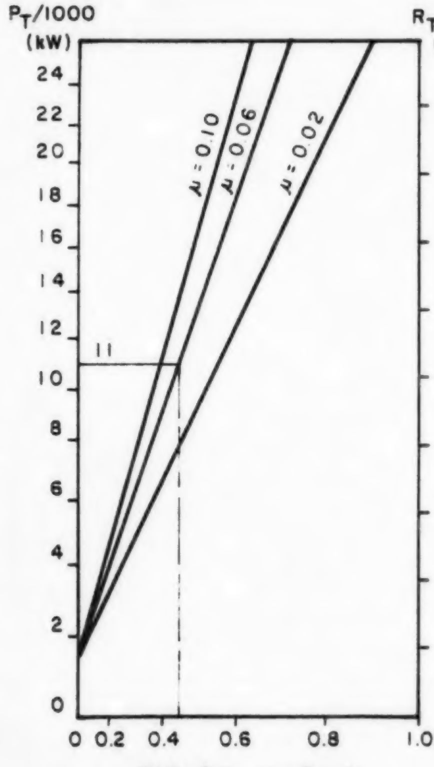

FIG. 8C V = 5m/s

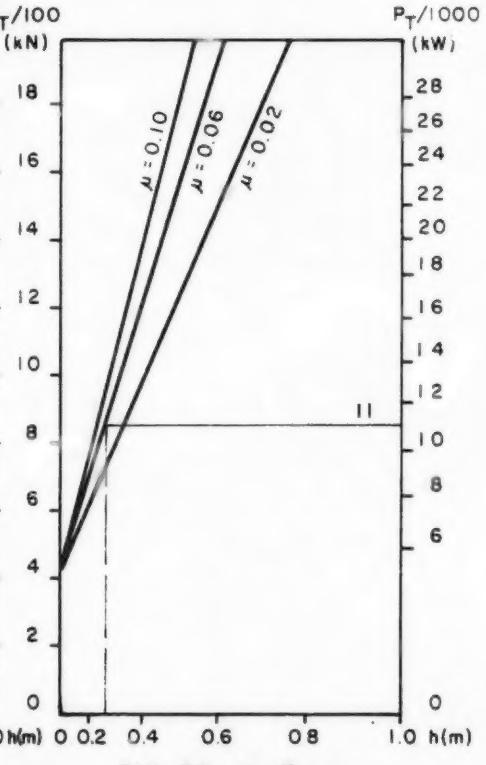

FIG. 8D V = 7m s

M.V. ARCTIC LEVEL ICE TOTAL RESISTANCE AND POWER REQUIREMENTS

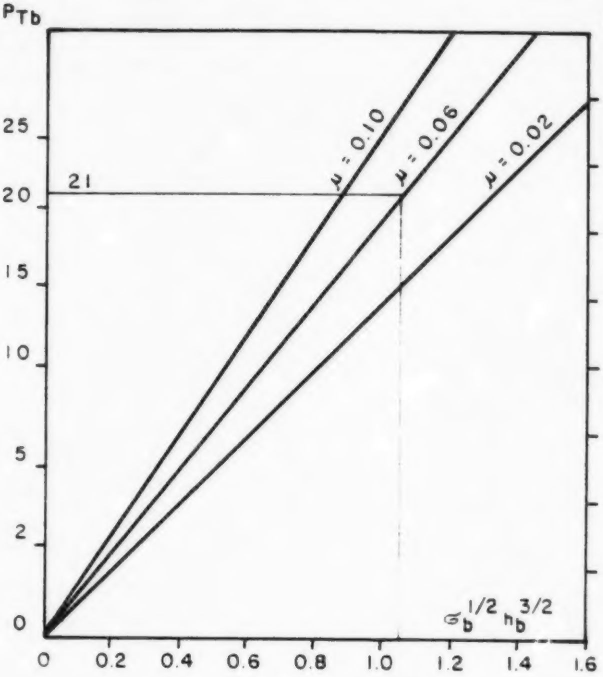

FIG. 9A $V_b = 0.2$

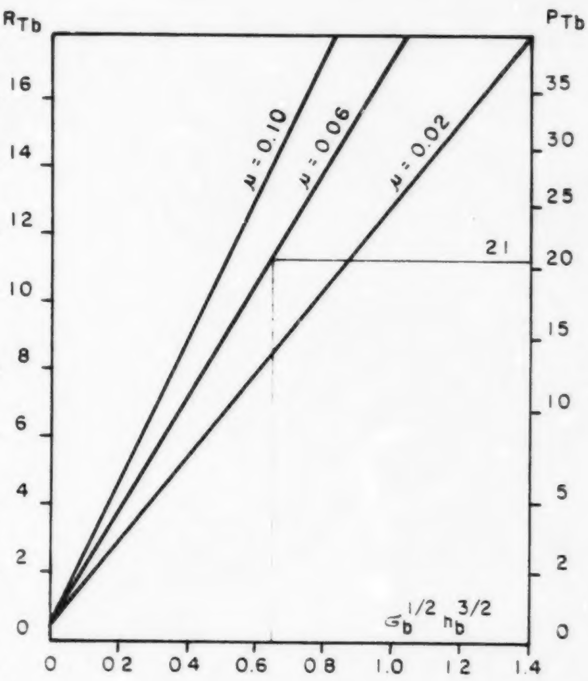

FIG. 9B $V_b = 0.6$

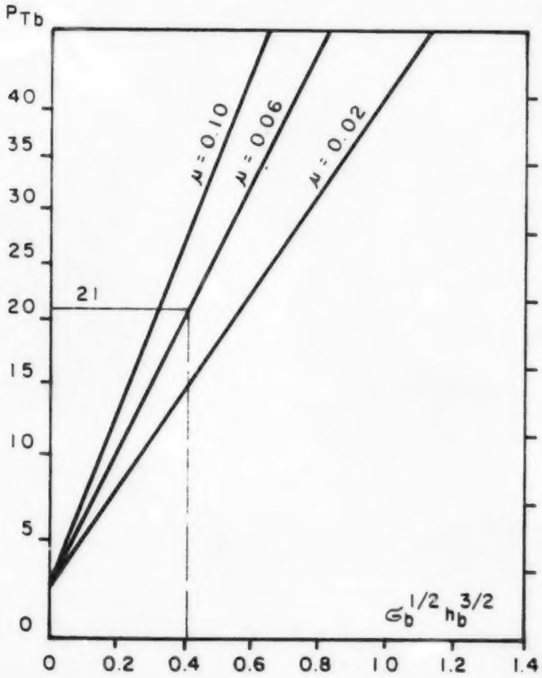

FIG. 9C $V_b = 1.0$

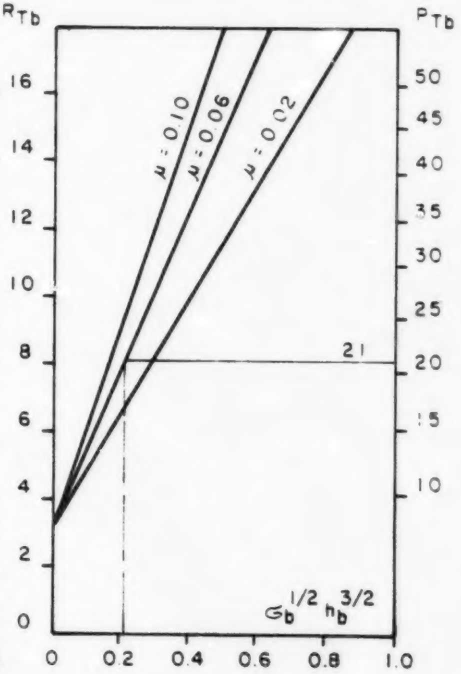

FIG. 9D $V_b = 1.4$

M.V. ARCTIC LEVEL ICE TOTAL RESISTANCE AND POWER COEFFICIENTS